MAT220 Test 2: Vectors and Vector-valued functions

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1 (10 pts). Given the equation of the plane

$$6x + 2y + 3z = 6$$

1. (6 points) Draw the plane in a suitable three dimensional coordinate system. Clearly indicate the coordinate axes x, y, and z, and the unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$.

2. (2 points) Find a point on the plane, and indicate its location in your drawing.

3. (2 points) Find a unit vector perpendicular to this plane, and draw it extending from the point you chose in part 2.

Problem 2 (10 pts). A few equations:

1. (5 points) Write the equation of the sphere of radius 4 with center (3,1,6).

2. (5 points) Write an equation for the plane perpendicular to the xy-plane passing through the points (0,0,1) and (1,1,1).

Problem 3 (20 pts).

1. (8 points) Draw the space curve with parametric equations given by

$$x(t) = \cos(2t)\cos(t) \quad y(t) = \sin(2t)\cos(t) \quad z(t) = \sin(t)$$

   from an eye position sighting along the vector $\mathbf{r} = \langle 1, 1, 1 \rangle$ towards the origin (that is, projecting onto the vectors $\mathbf{u}_1 = \left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle$ and $\mathbf{u}_2 = \left\langle \frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right\rangle$).

2. (4 points) Show that the motion occurs on a sphere.

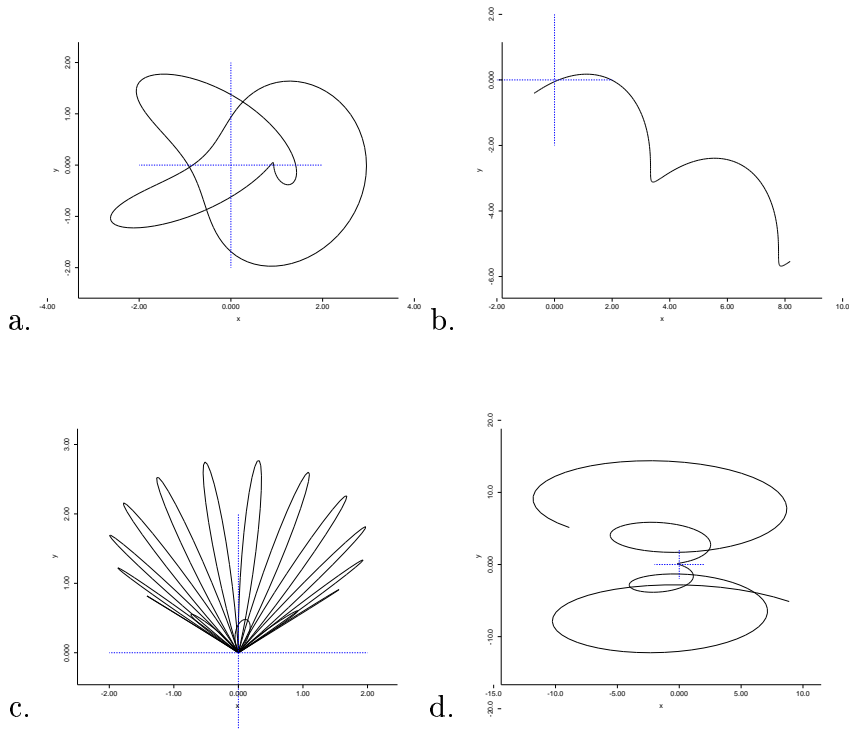3. (8 points) What is the equation of the line tangent to the curve when $t = \frac{\pi}{4}$?

Figure 1: Four space curves

Problem 4 (10 pts). Match the space curves in Figure 1 to the parametric equations given below by placing the correct letter in the correct matching box for the equations.

Notes: one of the sets of equations below has no corresponding plot! Leave its box empty.
   The curves are projected with the eye position along the vector $\mathbf{r} = \langle 1, 1, 1 \rangle$.

1. $\boxed{(\ \ )}$

$$x(t) = (2 + \cos(1.5t)) \cos(t) \quad y(t) = (2 + \cos(1.5t)) \sin(t) \quad z(t) = \sin(1.5t)$$

2. $\boxed{(\ \ )}$

$$x(t) = t \cos(t) \quad y(t) = t \sin(t) \quad z(t) = t$$

3. $\boxed{(\ \ )}$

$$x(t) = \cos(t) \quad y(t) = t \quad z(t) = \sin(t)$$

4. $\boxed{(\ \ )}$

$$x(t) = (1 + \cos(15t)) \cos(t) \quad y(t) = (1 + \cos(15t)) \sin(t) \quad z(t) = (1 + \cos(15t))$$

5. $\boxed{(\ \ )}$

$$x(t) = \sqrt{1 - 0.25 \cos(10t)} \cos(t) \quad y(t) = \sqrt{1 - 0.25 \cos(10t)} \sin(t) \quad z(t) = 0.5 \cos(10t)$$

Problem 5 (10 pts). Given $\mathbf{r_1} = \langle 0, 1, 3 \rangle$ and $\mathbf{r_2} = \langle -1, 2, 1 \rangle$. Compute:

1. $\mathbf{r_1} \cdot \mathbf{r_1} =$

2. $\mathbf{r_1} \cdot \mathbf{r_2} =$

3. $|\mathbf{r_1}| =$

4. $\mathbf{r_1} \times \mathbf{r_1} =$

5. $\mathbf{r_1} \times \mathbf{r_2} =$

Problem 6 (10 pts). A particle is accelerating with $\mathbf{a}(t) = \langle -\sin t, 0, -\cos t \rangle$. Suppose that its initial velocity is given by $\mathbf{v}(0) = \langle 1, 0, 0 \rangle$ and it is located at $\mathbf{r}(0) = \langle 0, 0, 1 \rangle$ initially.

1. (6 points) Find the vector function of position $\mathbf{r}(t)$.

2. (4 points) Describe the motion. What are the tangential and normal components of the acceleration?

Problem 7 (10 pts).
   Find the point at which the line

$$x = 1 + t \quad y = 2t \quad z = 3t$$

intersects the plane

$$x + y + z = 1.$$

Problem 8 (10 pts).

1. (5 points) What is the meaning of curvature? What does large curvature mean for a space curve? Small curvature? Can you draw a picture to make its meaning clear?

2. (5 points) Given $\mathbf{r}(t) = \langle t^2, 1, t \rangle$. Compute the curvature using the formula

$$\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}.$$

Problem 9 (10 pts). Determine whether the following expressions make sense or not. Indicate either N (nonsense), S (scalar), or V (vector) in the box at left.

1. ( ) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$

2. ( ) $\mathbf{a} \cdot (\mathbf{b} \cdot \mathbf{c})$

3. ( ) $|\mathbf{a}|(\mathbf{b} \cdot \mathbf{c})$

4. ( ) $\mathbf{a} \cdot \mathbf{b} + \mathbf{c}$

5. ( ) $\mathbf{a} \times \mathbf{b} + \mathbf{c}$