MAT385 Test 1: Chapters 1 and 2

Name:

Directions:

- All problems are equally weighted (8 points each).
- Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning).
- Indicate clearly your answer to each problem (e.g., put a box around it).
- **Good luck!**

Problem 1.
a. Verify that $A \to B$ is equivalent to $A' \vee B$.

b. Using part a, write the negation of the statement "If I study well, then I get a good grade."

Problem 2. Using propositional logic, prove that the following argument is valid:

"If the program is well-written, then it executes correctly. Either the program is well-written, or it is a disaster. Unfortunately, the program doesn't execute correctly. Therefore it is a disaster." (Use statement letters W, C, D.)

Problem 3. Using the predicate symbols shown and appropriate quantifiers, write each English language statement as a predicate wff. (The domain is the whole world.)

- L(x,y) - x loathes y
- S(x) - x is a student
- T(x) - x is a teacher

1. No student loathes all teachers.

2. Some student loathes some teacher.

Problem 4. Prove that the following argument is valid:

$$(\forall x)P(x) \wedge (\exists x)Q(x) \rightarrow (\exists x)[P(x) \wedge Q(x)]$$

Problem 5. The Fibonacci numbers are defined as follows:

$$\begin{aligned} &F(1) = 1 \\ &F(2) = 1 \\ &F(n) = F(n-1) + F(n-2) \end{aligned}$$

where $n \geq 3$. Show that the Fibonacci numbers satisfy the property

$$[F(1)]^2 + [F(2)]^2 + [F(3)]^2 + \ldots + [F(n)]^2 = F(n)F(n+1)$$

Problem 6. Prove that if n is an integer and n^2 is odd, then n is odd.

Problem 7. Use the "expand, guess, and check" method to find the closed-form solution to the linear, constant coefficient recurrence relation

$$\begin{aligned} S(1) &= a \\ S(n) &= bS(n-1) + c \end{aligned}$$

for n an integer with $n > 1$.

Problem 8. True or False?

1. ( ) In prolog an argument is proven by turning it into a disjunction, then exhaustively searching the database and applying *resolution* (essentially disjunctive syllogism) to the wff.

2. ( ) The deduction method allows us to prove an implication by padding the hypotheses with the consequent of the implication.

3. ( ) The second principle of mathematical induction is more powerful than the first principle.

4. ( ) $[B' \wedge (A \to B)] \to A'$

Extra Credit (4pts): prove that from a contradiction, anything follows.