MAT385 Test 2: Chapters 3.1, 5, 6, and 7.1

Name:

Directions:

- All problems are equally weighted (10 points each).
- Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning).
- Indicate clearly your answer to each problem (e.g., put a box around it).
- **Good luck!**

Problem 1. Consider the following two cases of purported Boolean algebras. Are they, or are they not? Explain!

a. B is the set of four elements $\{0, 1, a, b\}$, with b the inverse of itself.

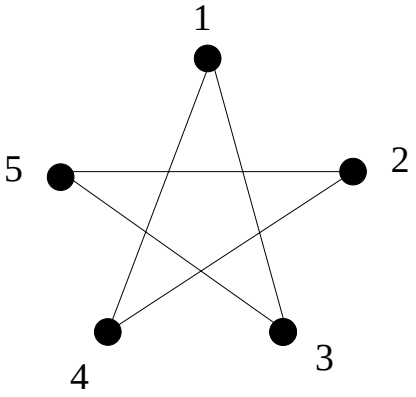
b. Consider the set $B = \{T, F\}$, the binary operation $\vee$ and $\wedge$, and the unary operation $'$ given by

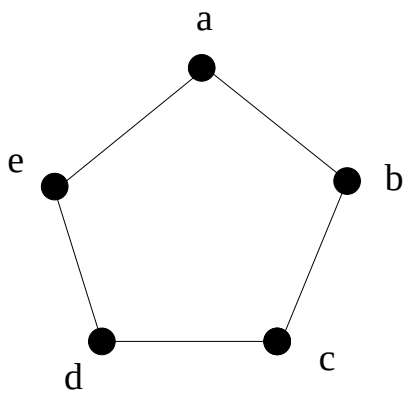
$\vee$	T	F
T	T	T
F	T	F

$\wedge$	T	F
T	T	F
F	F	F

$'$	
T	F
F	T

Problem 2. Determine whether the two graphs below are isomorphic. If so, prove it; if not, explain why not.





Problem 3. Consider the following subsets of the set of all animals:

- A - set of all mammals
- B - set of all reptiles
- C - set of all vertibrates
- D - set of all males

Using set operations, describe each of the following sets in terms of A, B, C, and D:

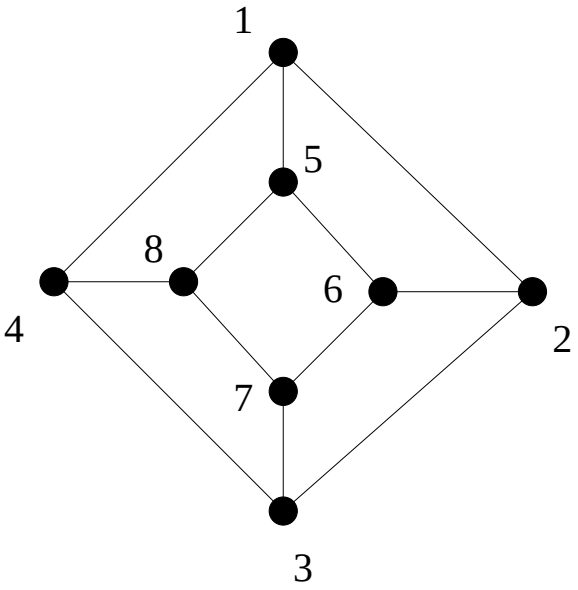
1. set of all female mammals

2. set of all female animals which are not reptiles

3. set of all vertibrates which are not mammals

4. set of all animals which are female or vertibrates

Problem 4. Consider the following graph:



Determine whether

1. An Euler path exists, and

2. Whether a Hamiltonian circuit exists.

Problem 5. The data $\{9, 12, 10, 5, 8, 2, 14\}$ is to be entered into a binary search tree.

1. Enter the data in the order shown, and draw the binary search tree.

2. Find another order for the data entry into a binary search tree which results in a tree of maximal depth.

Problem 6. You are to consider the graph represented by the following adjacency matrix:

$$\begin{bmatrix} 0 & 3 & 5 & \infty & 8 & 1 & \infty & \infty \\ 3 & 0 & 2 & \infty & \infty & \infty & 1 & \infty \\ 5 & 2 & 0 & 1 & \infty & \infty & \infty & 2 \\ \infty & \infty & 1 & 0 & 4 & \infty & \infty & \infty \\ 8 & \infty & \infty & 4 & 0 & 6 & \infty & 1 \\ 1 & \infty & \infty & \infty & 6 & 0 & 5 & \infty \\ \infty & 1 & \infty & \infty & \infty & 5 & 0 & 1 \\ \infty & \infty & 2 & \infty & 1 & \infty & 1 & 0 \end{bmatrix}$$

and suppose that we seek the shortest paths from node 3.

1. Use Bellman-Ford (AnotherShortestPath) to find the initial values of d and s,

	1	2	3	4	5	6	7	8
d								
s								

2. and then their values after the first iteration:

	1	2	3	4	5	6	7	8
d								
s								

Problem 7.

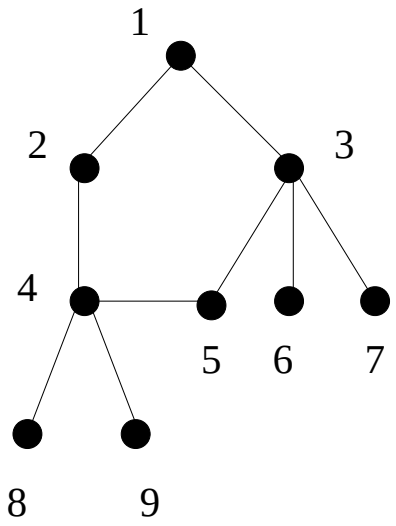
- Draw the graph corresponding to the adjacency matrix of Problem 6.

- Which nodes would be settled after two applications of Dijkstra's algorithm, if we were to start from node 1?

Problem 8. True or False?

1. [( )] A binary tree of depth d has at most 2^d leaves.
2. [( )] Euler's Formula for simple, connected planer graphs means that the addition of a couple of nodes to a graph must result in the increase in the number of edges.
3. [( )] A tree may be cyclic, as we see, for example, in the case of symbolic links in the UNIX file system tree.
4. [( )] A minimal spanning tree is unique.

Problem 9. Carry out both a depth-first and breadth-first traversal of the tree in the following figure, in each case starting from node 4:



Problem 10. For the tree in the following figure, write the inorder and the postorder traversals of the tree: