# MAT385 Final, Spring 2002

Name:

Directions:

- All problems are equally weighted. **You must skip one of problems 1-8, but you may not skip problems 9-11!** Write "SKIP" clearly on this problem.

- Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning).

- Indicate clearly your answer to each problem (e.g., put a box around it).

- **Good luck!**

Problem 1. True or False?

1. ( ) Assuming the statements P and P', we can prove anything.

2. ( ) The rule of thumb for proving predicate wffs is "universal instantiation before existential instantiation".

3. ( ) The bipartite complete graphs $K(m,n)$ are complete graphs.

4. ( ) A Hamiltonian Circuit is a cycle using every node of the graph.

5. ( ) A Finite-State Machine Recognizes a set of inputs S if the machine ends in a final state for every element of S.

Problem 2. Prove that the two graphs shown are isomorphic:
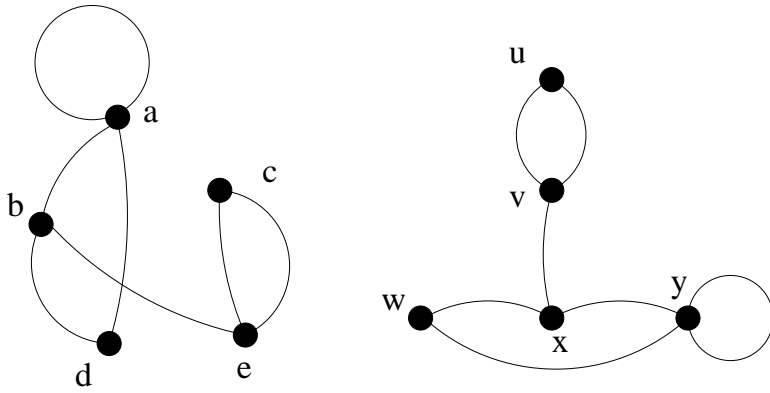
Problem 3. Using propositional logic, prove that the following argument is valid: "If fish mercury levels drop, then general health will improve. Either mercury emissions will drop, or general health will not improve. Fish mercury levels are dropping. Therefore, mercury emissions will drop." Use the statement letters F, G, M.

Problem 4. Consider the set S given by all strings composed of 0s and 1s where the number of 0s is a multiple of 2.

1. Represent set S as a regular expression (3 pts).

2. Construct a finite-state machine (whose input and output alphabets are the set $\{0, 1\}$) to recognize the regular set S (7 pts).

Problem 5. Using the predicate symbols shown and appropriate quantifiers, write each English language statement as a predicate wff. (The domain is the whole world.)

- S(x) is "x is a spy novel."

- L(x) is "x is long."

- M(x) is "x is a mystery."

- B(x,y) is "x is better than y."

1. Some mysteries are better than all spy novels.

2. Not every mystery is a spy novel.

3. Only mysteries are long.

4. Only spy novels are better than mysteries.

Problem 6. Use Floyd's algorithm to find the shortest distances between all pairs of points in the following graph. Show the intermediate matrices as you step through the algorithm.
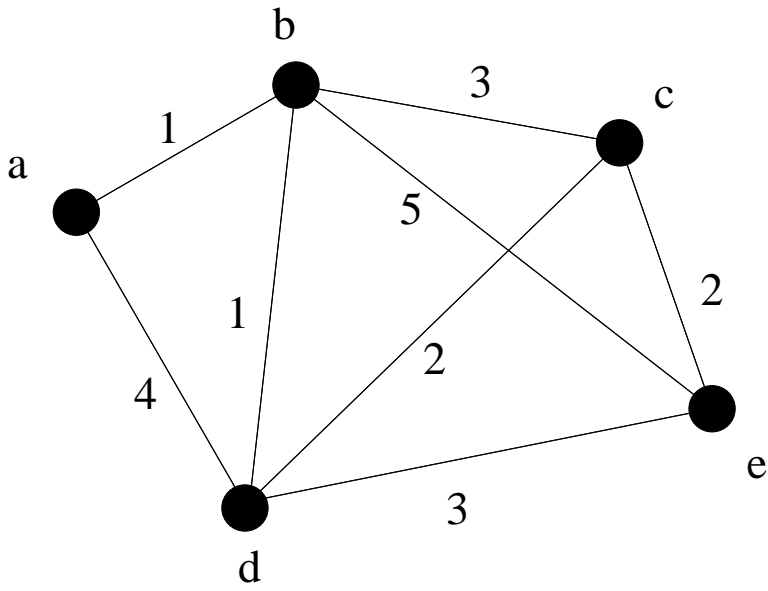
Problem 7. Solve the recurrence relation

$$C(n) = 1 + C(n/2)$$

for $n \geq 2$, $n = 2^m$,

- by the guess and check method, and then

- prove it by induction.

Note that $n = 2^m \iff m = \log_2 n$.

Problem 8. Given the infix expression $[(x - 5) * 7] + (5/3)$.

1. Draw the corresponding expression tree (6 pts).

2. Give the equivalent postfix and prefix expressions (4 pts):

   - postfix

   - prefix

Problem 9. Consider the set $B = \{T, F\}$, the binary operation $\vee$ and $\wedge$, and the unary operation $'$ given by

$\vee$	T	F
T	T	T
F	T	F

$\wedge$	T	F
T	T	F
F	F	F

$'$	
T	F
F	T

Prove that $[B, \vee, \wedge, ', F, T]$ is a Boolean algebra.

Problem 10. Given the following truth function:

x_1	x_2	x_3	x_4	$f(x_1, x_2, x_3, x_4)$
1	1	1	1	1
1	1	1	0	1
1	1	0	1	0
1	1	0	0	0
1	0	1	1	1
1	0	1	0	0
1	0	0	1	0
1	0	0	0	0
0	1	1	1	1
0	1	1	0	1
0	1	0	1	0
0	1	0	0	0
0	0	1	1	1
0	0	1	0	0
0	0	0	1	0
0	0	0	0	0

1. Find the canonical sum of products representation for this truth table (2 pts).

2. Draw the Karnaugh map (3 pts).

3. Use the Karnaugh map to minimize this truth function (5 pts). You should find the simplest equivalent Boolean expression.

Problem 11. Minimize machine M using the notion of k-equivalency, where M is given by

Present State	Next State Present Input 0	Next State Present Input 1	Output
0	5	3	1
1	5	2	0
2	1	3	0
3	2	4	1
4	2	0	1
5	1	4	0
6	2	3	1