MAT385 Test 1: Chapters 1 and 2

Name:

Directions:

- All problems are equally weighted.

- Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning).

- Indicate clearly your answer to each problem (e.g., put a box around it).

- **Good luck!**

Problem 1. Prove the following by induction:

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2} \text{ for } n \geq 1$$

Problem 2. Given the following Prolog rules relating to Me and my Dad.

```
child(Dad,Me)
child(Dad,Bill)
child(Sally,Bill)
child(Me,Bob)
child(Mary,Bob)
child(Mary,Sally)
spouse(Me,Mary)
spouse(Sally,Dad)
```

1. Define the recursive command *ancestor-of*, based on the rule *parent-of* which is defined as follows:
$$parent\text{-}of(X,Y) \textbf{ if } child(X,Y) \textbf{ or } (spouse(X,Z) \textbf{ and } child(Z,Y))$$

2. Trace execution of the command *ancestor-of(Me,X)*

Problem 3. Using propositional logic, prove that the following argument is valid. Use the statment letters H (students do homework), T (students do well on the test), C (students do well in the course).

The students have done their homework. Students don't do well on the test only if they haven't done their homework. If students do well on the test, then they will do well in the course. Therefore, students will do well in the course.

Problem 4. Prove that the pseudocode program segment is correct by finding the loop invariant Q, proving it, and evaluating Q at loop termination.

```
Factorial(positive integer x)
Local variables:
integers i,j
    i=2
    j=1
    while i neq x+1 do
        j=j*i
        i=i+1
    end while
    // j now has the value x!
    return j
end function Factorial
```

Problem 5. The "backward Fibonacci" numbers could be defined as follows for integer $n < 1$ $(n = 0, -1, -2, -3, \ldots)$:

$$BF(2) = 1$$
$$BF(1) = 1$$
$$BF(n) = BF(n+2) - BF(n+1)$$

1. Write the first eight terms of the sequence starting from $n = -1$ (remember, n goes backwards now)!

2. What is the relationship between the "forward" and "backward" Fibonacci sequences?

3. Prove that the relationship you've detected in part 2 above is correct.

Problem 6. Using predicate logic, prove that the following argument is valid:

$$(\forall x)(M(x) \rightarrow I(x) \vee G(x)) \wedge (\forall x)(G(x) \wedge L(x) \rightarrow F(x)) \wedge (I(j))' \wedge L(j) \rightarrow (M(j) \rightarrow F(j))$$

Problem 7. True or False?

1. [( )]   In a proof of correctness for a loop, the loop invariant Q is true before, during and after loop execution.

2. [( )]   DeMorgan's laws state that $(P \wedge Q)' = P' \wedge Q'$, and that $(P \vee Q)' = P' \vee Q'$.

3. [( )]   A Horn Clause appears in proof of correctness, as a pre-condition, code, and a post-condition of the form $\{Q\}P\{R\}$.

4. [( )]   The following is a first-order, constant coefficient, linear recurrence relation:

$$S(1) = a$$
$$S(n) = nS(n-1) + g(n)$$

5. [( )]   A proof can consist of merely testing a number (even a very large number!) of cases.