

Problem 2. Prove that a full binary tree of depth k has 2^k leaves, for $k \geq 1$.

Problem 3.

1. How many non-isomorphic simple graphs with 2 nodes are there?
2. How many non-isomorphic simple graphs with 3 nodes are there? (and draw all of them!).
3. How many non-isomorphic simple graphs with 4 nodes are there?

Problem 4. One of eight “Saddams” is the real thing. The question is, how many DNA tests must be performed to find the real Saddam?

Assume that if his DNA is in a sample, then the test will find it.

1. What is the number of leaves in the decision tree?
2. Find a lower bound on the number of tests required to find him in the worst case.
3. Describe a procedure that meets the lower bound in the worst case.

Problem 5.

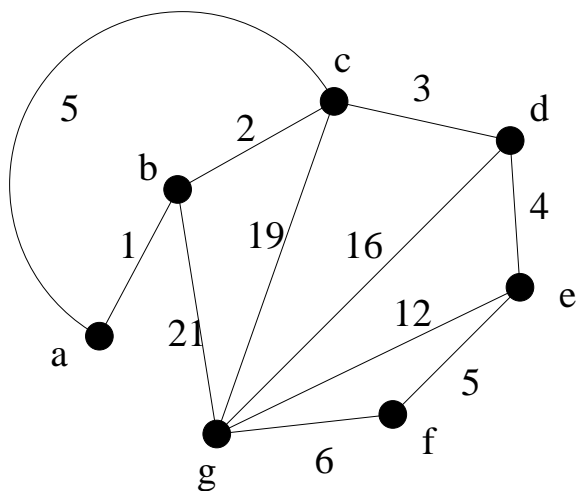
Given the expression $8/((4 - x) * 7 - y) + 15$ write the corresponding expression tree, then traverse the tree in pre- and post-order.

Problem 6. For which bipartite complete graphs will

- Euler paths exist?

- Hamiltonian circuits exist?

Problem 7. For the graph in the figure, use Dijkstra's algorithm to find the shortest path between



nodes a and g .

Problem 8. Give recursive definitions for the following:

1. The factorial function $n!$ for $n \geq 1$;

2. The Fibonacci numbers $F(n)$.

Formally characterize the recurrence relations in each case.

Problem 9. Consider the set S of all binary strings containing the substring “010”. Construct a finite-state machine (whose input and output alphabets are the set $\{0, 1\}$) to recognize the regular set S . Try to be neat! A minimal machine will result in all points; “more than minimal” will not!

Problem 10. Consider the following truth function:

x_1	x_2	x_3	$f(x_1, x_2, x_3)$
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	1

1. Write the corresponding canonical sum of products.
2. Use the Karnaugh map to minimize the boolean expression in part 1.
3. Use Quine-McKluskey to minimize the boolean expression in part 1.

Problem 11. Write regular expressions for the following regular sets over the alphabet $\{0,1\}$:

- Finite strings containing three 1s.

- Finite strings containing at least three 1s.

- Finite strings containing at most three 1s.

- Finite strings containing multiples of three 1s.

Problem 12. Minimize the finite state machine given by the following state table, and draw the state graph of the minimized machine:

Table 1: An FSM cries out for minimization

Present State	Next state		output
	0	1	
0	3	5	1
1	1	6	1
2	0	4	0
3	1	6	1
4	5	3	0
5	4	1	0
6	2	5	1

Problem 13. A nuclear device will only activate if at least three of four keys x_1 , x_2 , x_3 , and x_4 are turned “on”. You are to design the machine that assures no unwanted nuclear winters.... Carry out this task:

1. Find a truth function,
2. a canonical sum of products,
3. minimize the canonical sum of products, and
4. build a logic network of the minimal expression above.