## MAT385 Final, Fall 2003

## Name:

## Directions:

- All problems are equally weighted. You must skip three of problems 1-8, but you may not skip both problems 1 and 2! Write "SKIP" clearly on these problems.
- Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning).
- Indicate clearly your answer to each problem (e.g., put a box around it).
- Good luck!

## Problem 1.

• Use propositional logic to prove the following argument valid: If the roads are wet or icy, then driving is treacherous. Accidents will occur when driving is treacherous. The roads are icy. Therefore accidents will occur.

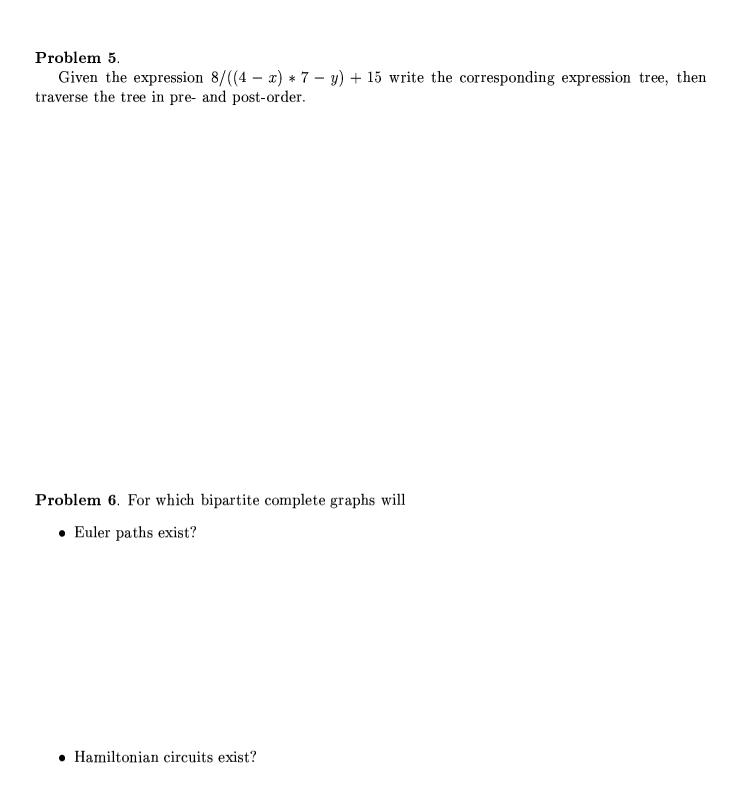
• Use predicate logic to prove the following argument valid: All tests are unpleasant; some tests are hard. Therefore there is a hard, unpleasant test.

Prol	olen	n 2.	Pro	ve th	ıat a	ı ful	l bi	nary	y tr	ee o	of d	ept.	h k	has	$5 2^k$	leav	ves, f	for i	$k \ge 1$				
Prol																							
1.	Но	w ma	"ny	non-	isom	orp	hic	$\sin$	ple	gra	aphs	s wi	th :	2 no	odes	are	the	re?					
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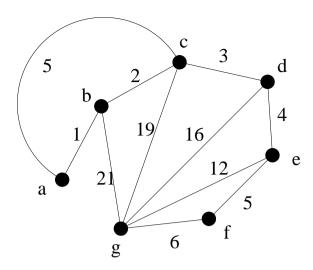
**Problem 4**. One of eight "Saddams" is the real thing. The question is, how many DNA tests must be performed to find the real Saddam?

Assume that if his DNA is in a sample, then the test will find it.

- 1. What is the number of leaves in the decision tree?
- 2. Find a lower bound on the number of tests required to find him in the worst case.
- 3. Describe a procedure that meets the lower bound in the worst case.



Problem 7. For the graph in the figure, use Dijkstra's algorithm to find the shortest path between



nodes a and g.

Problem 8. Give recursive definitions for the following:

1. The factorial function n! for  $n \ge 1$ ;

2. The Fibonacci numbers F(n).

Formally characterize the recurrence relations in each case.

**Problem 9**. Consider the set S of all binary strings containing the substring "010". Construct a finite-state machine (whose input and output alphabets are the set  $\{0,1\}$ ) to recognize the regular set S. Try to be neat! A minimal machine will result in all points; "more than minimal" will not!

**Problem 10**. Consider the following truth function:

$x_1$	$x_2$	$x_3$	$f(x_1, x_2, x_3)$
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	1

- 1. Write the corresponding canonical sum of products.
- 2. Use the Karnough map to minimize the boolean expression in part 1.

3. Use Quine-McKluskey to minimize the boolean expression in part 1.

<b>Problem 11</b> . Write regular expressions for the following regular sets over the alphabet $\{0,1\}$ :
• Finite strings containing three 1s.
• Finite strings containing at least three 1s.
• Finite strings containing at most three 1s.
• Finite strings containing multiples of three 1s.

**Problem 12**. Minimize the finite state machine given by the following state table, and draw the state graph of the minimized machine:

Table 1: An FSM cries out for minimization

Present	Next	state	output
State	0	1	
0	3	5	1
1	1	6	1
2	0	4	0
3	1	6	1
4	5	3	0
5	4	1	0
6	2	5	1

**Problem 13**. A nuclear device will only activitate if at least three of four keys  $x_1$ ,  $x_2$ ,  $x_3$ , and  $x_4$  are turned "on". You are to design the machine that assures no <u>unwanted</u> nuclear winters.... Carry out this task:

- 1. Find a truth function,
- 2. a canonical sum of products,
- 3. minimize the canonical sum of products, and
- 4. build a logic network of the minimal expression above.