MAT385 Test 1: Chapters 1 and 2

Name:

Directions:

- All problems are equally weighted.
- Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning).
- Indicate clearly your answer to each problem (e.g., put a box around it).
- **Good luck!**

Problem 1. Prove or disprove:

1. (5pts) The sum of three consecutive integers is divisible by 3.

2. (5pts) The sum of any five consecutive integers is even.

Problem 2. Given the following Prolog rules relating to courses and outcomes in the Department of Math and CS:

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pre-req-of(degree, mat385)
pre-req-of(mat385, csc262)
pre-req-of(mat385, mat220)
pre-req-of(mat220, mat120)
pre-req-of(csc262, csc260)
pre-req-of(mat120, mat119)
pre-req-of(csc260, mat119)
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By pre-req-of(csc260, mat119), we mean that "a pre-requisite of csc260 is mat119".

1. (4pts) Define the recursive command *needed-before*, based on *pre-req-of*, which would be used to indicate that "needed before X is Y" using the syntax *needed-before*(X, Y).

2. (5pts) Trace execution of the command *needed-before*$(degree, Y)$

3. (1pts) Introduce a datum that would result in an infinite loop in the command above.

Problem 3. Prove the following using propositional logic:

1. (5pts) If the program is bad, the graduates do poorly. Either the program is bad, or it has many good qualities. The graduates do well. Therefore the program has many good qualities.

   (use statement letters B(ad), P(oorly), and Q(ualities)).

2. (5pts)

$$(A' \to B') \wedge (A \to C) \to (B \to C)$$

Problem 4. Suppose we alter algorithm *BinarySearch* slightly to create *TrinarySearch*, as follows: rather than splitting the sorted list into two "equal" parts and checking the middle element, we split it into three, and check the "two middle elements" (wedged between the three sublists).

In order to perform an analysis of the algorithm, we will count comparisons as the search progresses, in the worst-case scenario of the element not belonging to the list.

1. (5pts) Write a linear recurrence relation that gives the number of comparisons $C(n)$ at step n in terms of the number of comparisons C of some sublist(s).

2. (5pts) Rather than using $n = 2^m - 1$ to assure that the split always occurs symmetrically, as we discovered that we should do in the case of *BinarySearch*, what value of n should be chosen so as to make sure that we can "divide and conquer" successfully at each step, down to the base case C(1), ensuring that all sublists are equal in size? **Justify your answer!** You may want to draw diagrams. Hint: what would be the size of the list successfully split down to a size of 1?

Problem 5. The "Newinacci" numbers are defined as follows:

$$S(1) = 1$$
$$S(2) = 2$$
$$S(n) = S(n-1) \cdot S(n-2)$$

for integer $n \geq 3$.

1. (2pts) Write the terms through S(7).

2. (4pts) Find a closed form solution for the Newinacci numbers in terms of other functions that you already know.

3. (4pts) Prove that the closed form solution in part 2 above is correct.

Problem 6. Consider the following disagreeable argument:

$$(\forall x)[P(x) \vee Q(x)] \wedge (\exists x)[Q(x)]' \rightarrow (\exists x)Q(x) \vee (\forall x)P(x)$$

1. (2pts) Explain in plain English (that my mother would understand) the sense of the argument. (My mother is not a mathematician!)

2. (2pts) Prove that $R \vee S$ and $R' \rightarrow S$ are equivalent wffs.

3. (6pts) Using predicate logic, prove that the disagreeable argument is valid.

Problem 7. Prove that $2^n > n^2$ for $n \geq 5$. What (correct) general conclusion might you infer about different function types?

Extra Credit (4pts): On the back of this page, prove the following rule for derivatives of positive integral power of x:

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

for all $n \geq 1$. You are given the definition of the derivative

$$\frac{d}{dx}(f(x)) = \lim_{h \longrightarrow 0} \frac{f(x+h) - f(x)}{h}$$

and the product rule

$$\frac{d}{dx}(f(x) \cdot g(x)) = \frac{d}{dx}(f(x)) \cdot g(x) + f(x) \cdot \frac{d}{dx}(g(x)).$$