MAT310 Test 1: Chapters 1-3

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Today's Special: you may skip one of problems 3-8 (not 1 or 2). Write "skip" on the one you do not want me to grade.

Problem 1 (10 pts) Prove (Division Algorithm): given integers a and b, with $b > 0$, there exist unique integers q and r satisfying

$$a = qb + r \qquad 0 \leq r < b.$$

Problem 2 (10 pts).

1. If 1 is added to a product of twin primes, prove that a perfect square is always obtained.
2. Show that the sum of twin primes p and $p+2$ is divisible by 12, provided that $p > 3$.

Problem 3 (10 pts). What is the largest integer that one would have to test to determine whether 1,000,000,000,061 is prime? Justify your answer: it is not enough to simply assert something – you must show why it is true.

Problem 4 (10 pts). Calculate gcd(1776, 2005) **using the Euclidean algorithm**, and express this gcd as a linear combination of 1776 and 2005.

Problem 5 (10 pts). Determine all solutions to the Diophantine equation $4x + 5y = 30$, and determine all solutions that are positive.

Problem 6 (10 pts). For any odd integer a, show that $gcd(3a, 3a + 2) = 1$.

Problem 7 (10 pts). For $n \geq 1$ prove that $n(n+1)(2n+1)/6$ is an integer. [Use cases!]

Problem 8 (10 pts). The integer n is a triangular number if and only if $8n+1$ is a perfect square.

Extra Credit (4 pts). What can you tell me about Gauss and Pythagoras? (I'm looking for stories from your reading, as well as stories I told....)