MAT385 Final, Spring 2005

Name:

Directions:

- All problems are equally weighted. **You must skip one of problems 1-7, but you may not skip problems 8-11!** Write "SKIP" clearly on the problem you skip.
- Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning).
- Indicate clearly your answer to each problem (e.g., put a box around it).
- **Good luck!**

Problem 1. True or False? If false, explain how to make the statement true.

1. [( )] If the class is a success, then learning will increase.
   Either the class is successful, or the school will close.
   The learning will not increase. Therefore the school will close.

2. [( )] The Bellman-Ford algorithm produces the shortest path between any two points in a graph.

3. [( )] One advantage of a preorder traversal of a tree is that the nodes can be deallocated as they are written.

4. [( )] Any machine can be represented as a Finite State Machine.

5. [( )] A Hamiltonian circuit is a an example of a graph traversal.

Problem 2. Short Answer:

- Write the regular expression for the regular set of binary strings containing either an even number of zeros or strings beginning with 1 and ending with 11.

- What are the steps of a proof by induction? What is the difference between the first and second principles of induction?

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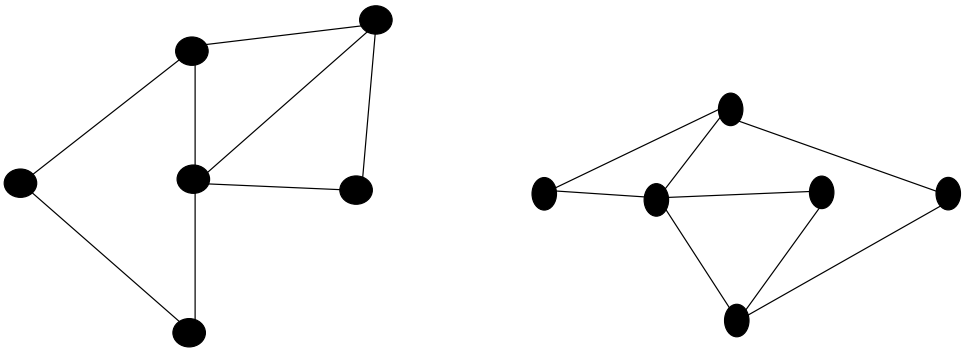


- Given a set A of n elements, what is the size of the power set of A?

Problem 3. One of the four "perfect syllogisms" of Aristotle (384-322 B.C.) is named "Celarent": Using predicate logic, write the syllogism as a predicate wff, and then prove it.

- No M are P.
- All S are M.
- Therefore, no S are P.

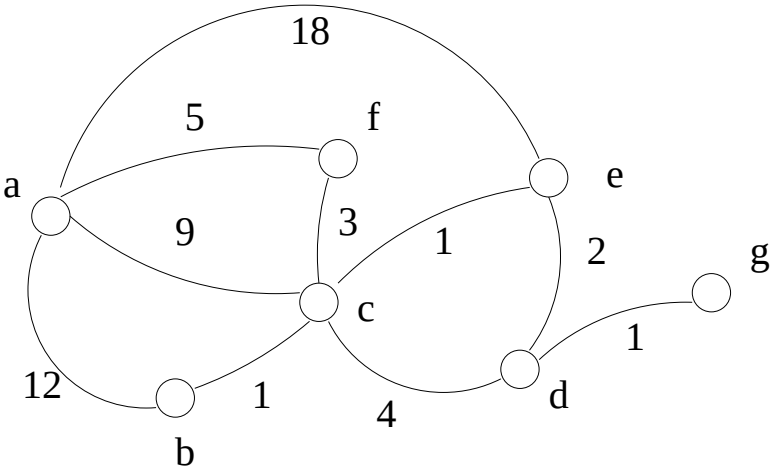
Problem 4. The names {shakya wang rogers narasapur nadaud knuckles rathje beyer mcmillan steveson weeks findley chirawu carter} are entered into a binary search tree. (In the following you should represent each name by the fewest letters needed to distinguish it from the other names.)

- Draw the tree. To the right of the tree, find the number of comparisons needed in a binary tree search if the name "long" is sought in the data.

- What is the maximal number of nodes in a binary tree of depth d?

- If you entered the data in an optimal way so as to minimize depth, what would be the depth of the tree? Give an ordering of the data that achieves this depth, and draw the tree.

Problem 5.



1. (8 points) Use Dijkstra's algorithm (in all its glory) to find the shortest distance from node a to g. Make sure to show the details as you step through the algorithm.

2. (2 points) This graph is planar: what relationship exists between the number of nodes, arcs, and regions for a planar graph?

Problem 6. Consider the equation relating Fibonacci numbers

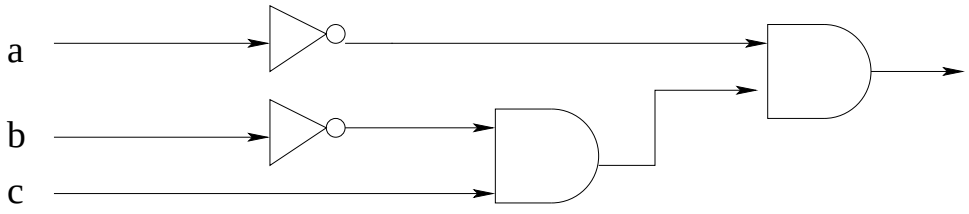
$$F(n+4) = F(n+2) + 2F(n+1) + F(n)$$

for $n \geq 1$.

1. Prove this by the definition of the Fibonacci numbers.

2. Prove it by induction.

Problem 7. Replace the following network with an equivalent network using one AND gate, one OR gate, and one inverter:



Problem 8. Draw the state graph for the following machine, and compute the output sequence for the input sequence 0011:

Table 1:

Present State	Next State		Output
	Present Input		
	0	1	
s_0	s_2	s_3	0
s_1	s_0	s_1	1
s_2	s_1	s_3	0
s_3	s_1	s_2	1

Problem 9. Construct a finite-state machine whose job is to recognize that a pair of lines from the Quine-McLuskey algorithm is a match, and should be combined to make a new line in the next table.

The input alphabet is $\{00, 11, --, 01, 10, -0, -1, 0-, 1-\}$, the set of possible pairs of digits from two lines.

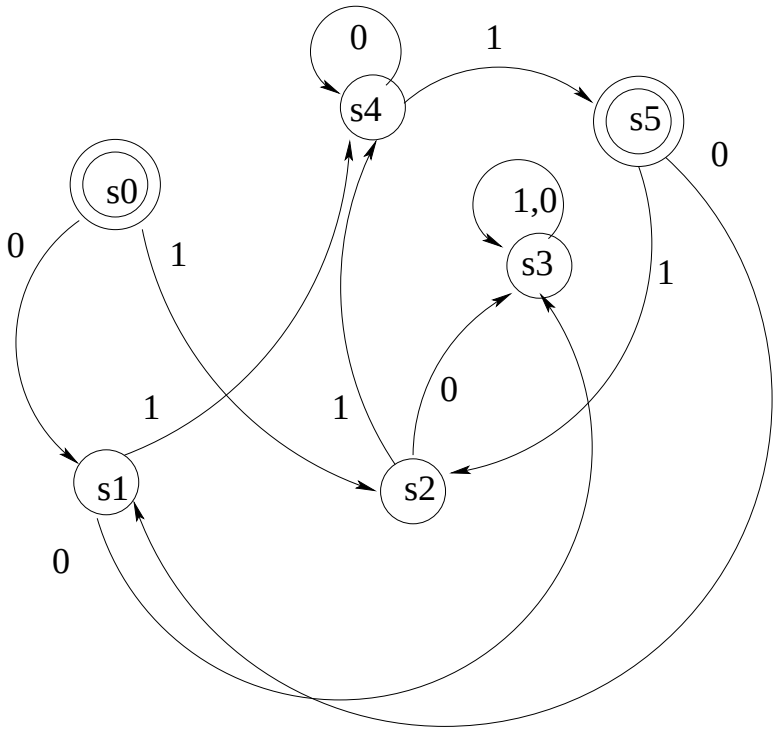
Problem 10. Consider the following truth function:

x_1	x_2	x_3	x_4	$f(x_1, x_2, x_3, x_4)$
1	1	1	1	1
1	1	1	0	1
1	1	0	1	1
1	1	0	0	1
1	0	1	1	0
1	0	1	0	1
1	0	0	1	0
1	0	0	0	1
0	1	1	1	1
0	1	1	0	0
0	1	0	1	1
0	1	0	0	1
0	0	1	1	0
0	0	1	0	0
0	0	0	1	1
0	0	0	0	1

1. Draw the Karnaugh map (3 pts).

2. Use the Quine-McCluskey algorithm to minimize the Boolean expression. [Be clever!]

Problem 11. Given the finite state machine of the following graph:



- Write the corresponding state table in the space to the right of the machine.

- Minimize the machine.