MAT385 Test 1: Chapters 1 and 2

Name:

Directions:

- All problems are equally weighted.
- Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning).
- Indicate clearly your answer to each problem (e.g., put a box around it).
- **Good luck!**

Problem 1.

1. (5pts) Decide whether the following is a Tautology (show your work!): $A \vee (A \wedge B)'$

2. (5pts) Negate the predicate wff
$$(\exists x)(\forall y)P(x, y)$$
and, in words, give an interpretation in which the given statement is true, and then an interpretation in which its negation is true.

Problem 2. Given the following Prolog rules relating to breakfast:

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ingredient-of(omelette,egg)
ingredient-of(omelette,tomato)
ingredient-of(omelette,onion)
ingredient-of(omelette,olive)
ingredient-of(pancake,flour)
ingredient-of(pancake,egg)
ingredient-of(pancake,oil)
ingredient-of(flour,wheat)
ingredient-of(oil,soybean)
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1. (2pts) Trace execution of the command *ingredient-of(X,egg)*

2. (4pts) Define the recursive command *required-before*, based on *ingredient-of*, which would be used to indicate that "needed before X is Y" using the syntax *required-before(X, Y)*.

3. (4pts) Trace execution of the command *required-before(pancake, Y)*

Problem 3. Prove or disprove the following using propositional logic:

1. (5pts) If Betty has bought a fur coat, then either she has robbed a bank or her rich uncle has died. Her rich uncle has not died. Therefore, if Betty has not robbed a bank, then she has not bought a fur coat.

   (use statement letters F(ur), R(obbed), and U(ncle)).

2. (5pts) If there is life on Mars, then the experts are wrong and the government is lying. If the government is lying, then the experts are right or there is no life on Mars. The government is lying. Therefore there is life on Mars.

   (use statement letters L(ife), E(xperts), and G(overnment)).

Problem 4. Using predicate logic and statement letters C(x), A(x), B(x,y), s, S(x), prove that the following argument is valid:

Every crocodile is bigger than every alligator. Sam is a crocodile. But there is a snake, and Sam isn't bigger than that snake. Therefore, something is not an alligator.

Problem 5. In class we used the "expand, guess, and check" method to find the closed form solution for the linear, first order, constant coefficient recurrence relation, given by

$$\begin{aligned} S(1) &= a \\ S(n) &= cS(n-1) + g(n) \end{aligned}$$

for n an integer with $n > 1$. We found that

$$S(n) = c^{n-1}a + \sum_{i=2}^{n} c^{n-i} g(i)$$

Use induction to prove that this is the correct closed form solution.

Problem 6. Pyramid schemes are built on the following sort of principle: You receive a copy of a letter making fabulous claims about how much money you can "earn" by participating. In the message is an ordered list of 5 people. You are asked to send \$5 to the first person on the list; you then alter the list of previous participants, removing the one at the top of the list, moving all the other names up one position, and adding your own name to the bottom of the list. You then send out 10 copies of this altered letter to others.

The five people running this sort of con get together and put their names on the list. They then send out lots of copies, and wait for the money to come rolling in. Assume that 1000 copies of the initial letter are sent out, and that everyone participates as requested.

- (4 pts) Write a linear recurrence relation giving the amount of money $M(n)$ that the people running the con (persons in positions 1-5) will receive, for $1 \leq n \leq 5$.

- (4 pts) How much can you expect to receive when your name gets to the top of the list?

- (2 pts) If no one participates more than once, and if everyone responds to the message one week after receiving the letter, how long before the number of letters surpasses the number of people on Earth (roughly 7 billion now)?

Problem 7. Prove that the sum of an integer and its cube is even.

Problem 8. True or False?

1. [( )] Prolog uses proof by exhaustion to resolve arguments.
2. [( )] Tautology is the predicate logic version of validity.
3. [( )] A proof of $P \longrightarrow Q$ by contraposition means proving $Q' \longrightarrow P'$.
4. [( )] In applying BinarySearch to the list $\{1, 2, 5, 8, 7, 9, 11, 13, 18, 19, 20\}$ for target x=6, x is never compared to 5.

Extra Credit (4pts): Prove that

$$\sin\theta + \sin 3\theta + \cdots + \sin(2n-1)\theta = \frac{\sin^2 n\theta}{\sin\theta}$$

for all $n \geq 1$ and for all θ for which $\sin\theta \neq 0$.