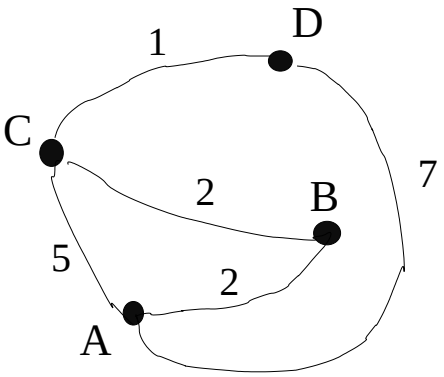
MAT385 Spring 2005, Test 2: Chapters 3.1, 5, 6, and 7.1-2

Name:

Directions:

- Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning).
- Indicate clearly your answer to each problem (e.g., put a box around it).
- **Good luck!**

Problem A (20 points). Given the graph below,



1. Write down an appropriate adjacency matrix representing the graph.

2. Write down an appropriate adjacency list representing the graph.

3. Use Dijkstra's algorithm to find the shortest distance from node a to d, and the corresponding paths.

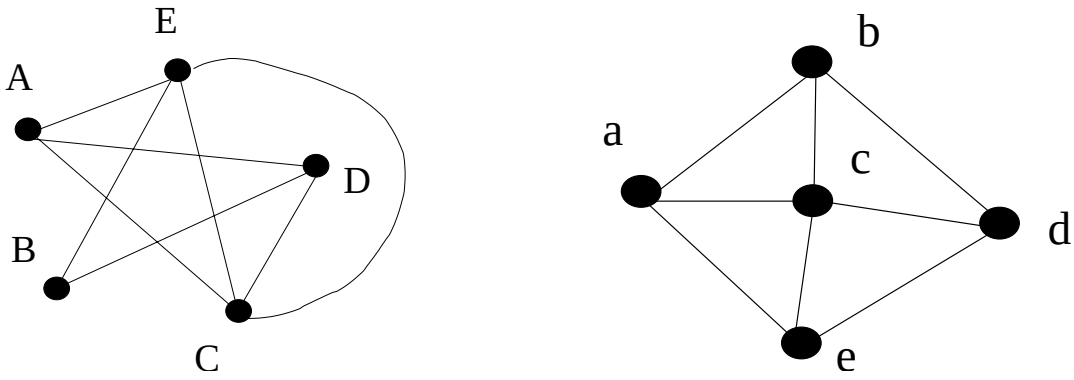
4. Use Bellman-Ford's algorithm to find the shortest distance from node a to every other node.

5. Use Floyd's algorithm to find the shortest distance between every pair of nodes.

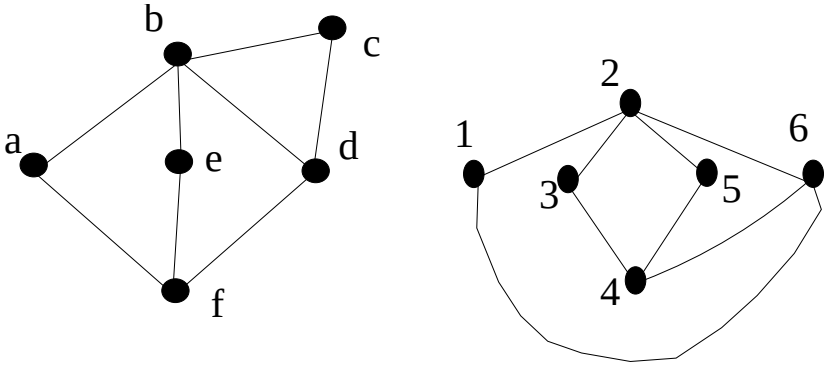
6. How many different Euler paths exist for this graph?

Problem B (10 points). For the following pairs of graphs, demonstrate conclusively whether they are, or are not, isomorphic:

Pair 1:



Pair 2:



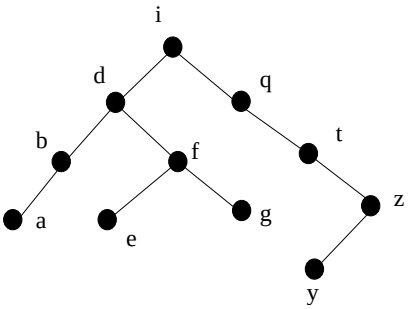
Problem C (10 points). A poor electrician wires two lights with three switches, as given in the following truth table/function:

s_1	s_2	s_3	l_1	l_2
1	1	1	1	0
1	1	0	0	1
1	0	1	0	1
1	0	0	1	0
0	1	1	0	1
0	1	0	1	0
0	0	1	1	0
0	0	0	0	0

1. Find the canonical sum of products representations for the two lights.

2. Draw the corresponding logic networks equivalent to the cannonical expressions.

Problem D (10 points). Consider the tree given in the following figure:



1. Order the data (set of letters) stored in the tree so that the binary search tree will correspond to the tree above.

2. What is the theoretical lower limit on the depth of a binary search tree of 11 data items?

3. Give an ordering of the data that achieves this lower limit; then given an ordering that achieves the worst case for depth.

4. Do a postorder and preorder traversal of the tree.

Problem E (10 points). Prove the following for all Boolean algebras:

1. $$x + (x \cdot y) = x$$

2. $$x \cdot (x + y) = x$$

3. $$x \cdot (z + y) + (x' + y)' = x$$

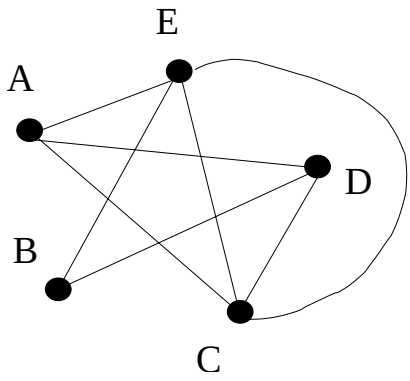
Problem F (10 points). For the set A containing the 11 data points of problem #4:

1. how many elements are in the power set of A?

2. Give four qualitatively different examples of elements of the power set of A.

3. How many distinctly different subsets of four elements can be constructed from A? (You might use a tree to help you count!)

Problem G (10 points). For the graph below, perform breadth-first and depth-first traversals,



starting from node A.

Extra Credit (4 points) Can any of the poor electrician's set of three switches be eliminated such that the lights still work?