MAT385 Final, Spring 2006

Name:

Directions:

- All problems are worth 10, except Problem 9 (which is worth 20). **You must skip one of problems 1-7, but you may not skip problems 8-10!** Write "SKIP" clearly on the problem you skip.
- Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning).
- Indicate clearly your answer to each problem (e.g., put a box around it).
- **Good luck!**

Problem 1. True or False? If false, explain how to make the statement true.

1. [( )] $(\exists x)(Boy(x) \to Girl(x))$, where the domain of x is every animal on earth. (where $Boy(x)$ signifies that x is a Boy, etc.)

2. [( )] A Logic Network is equivalent to a Truth Tables is equivalent to a Boolean Algebra.

3. [( )] The optimization of Hamiltonian circuits on weighted graphs goes by the name of "The Travelling Salesman Problem."

4. [( )] To prove $P \longrightarrow Q$ by contraposition, assume Q' and deduce P'.

5. [( )] Prim's and Kruskal's algorithm will produce identical minimal spanning trees.

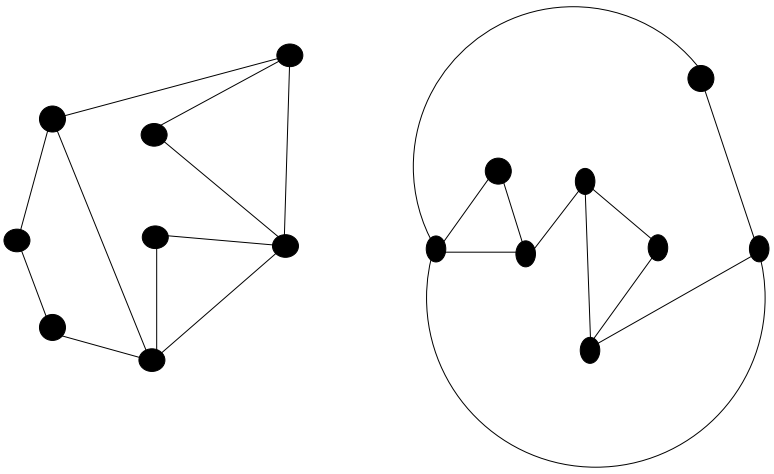
Problem 2. Short Answer:

- How many distinct Euler paths exist in K_4? (Paths are considered distinct if they have different orders of traversal.)

- Consider the set $A = \{\text{Barbara, Celarent, Darii, Ferio}\}$ of the four perfect syllogisms of Aristotle (384-322 B.C.). How many elements total are in the power set of A? Break down the elements of the power set by size of subset in the following table:

total	size 0	size 1	size 2	size 3	size 4

- Decide whether the following graphs are isomorphic or not, with reason(s).



- Simplify (assuming $p \neq 1$): $1 + p + p^2 + \ldots + p^{n-1} + p^n$

Problem 3. One of the four "perfect syllogisms" of Aristotle is named "Darii".

- Every B is A.
- Some C is B.
- Therefore, some C is A.

1. (2 pts) Give a practical everyday example of this syllogism in action.

2. (8 pts) Using predicate logic, write the syllogism as a predicate wff, and then prove it.

Problem 4. My friend Dan, Jr. had his first baby yesterday (Daniel the Third, born May 2, 2006). Dan's father (Dan, Sr.) has two children. Assuming that every one of his descendants survives to reproduce and has two children (at least one male), answer the following questions:

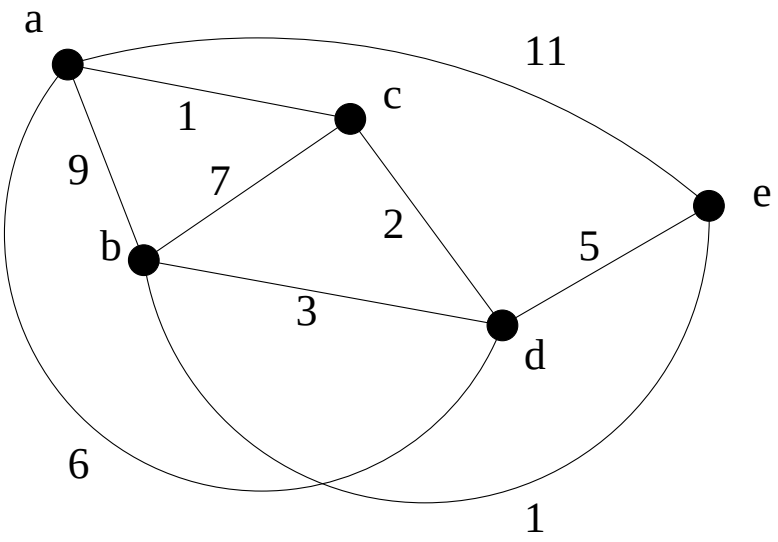
1. (4 pts) With Dan Sr. (Dan the First!) as the root of a tree of descendants, at what depth will Dan the 18^{th} appear (assuming that each Dan the i^{th} produces a Dan the $(i+1)^{th}$), and how many total descendants will join Dan the 18^{th} at this same depth?

2. (4 pts) Write a recurrence relation that provides the number of people $DF(d)$ at depth d in the tree described above ("DF" for "DanFolk"); give a closed form solution for the number of people at depth d.

3. (2 pts) Find a closed form solution for the total number of people in the tree, from Daniel the First to those at depth d:

$$\sum_{0}^{d} DF(i) =$$

Problem 5.



1. (8 points) Use the Bellman-Ford algorithm to find the shortest distance from node a every other node. In particular, give the optimal path from node a to node e.

Step 1:		a	b	c	d	e
	d					
	s					

Step 3:		a	b	c	d	e
	d					
	s					

Step 5:		a	b	c	d	e
	d					
	s					

Step 2:		a	b	c	d	e
	d					
	s					

Step 4:		a	b	c	d	e
	d					
	s					

Step 6:		a	b	c	d	e
	d					
	s					

2. (2 points) Most unfortunate, that crossing in the graph above: can the graph be made planar? If so, show how; if not, explain why not.

Problem 6.

1. Translate the following statements into an argument provable by propositional logic, and then prove it (use statement letters B - Bob got an F, W - World turning, F - Flown off):

   If Bob got an F, then the world would have stopped turning. If the world stopped turning, then we would have all flown off. We have not flown off the Earth. Therefore Bob didn't get an F.

2. Construct a binary tree using the letters in the word "brainstem" so that a preorder traversal will spell out the word.

Problem 7. Prove the following:

1. $$(x + y \cdot x)' = x'$$

2. The *Lucas sequence* is defined by $L(1) = 1$, $L(2) = 3$, and
$$L(n) = L(n-1) + L(n-2)$$
Prove that $L(n) = F(n) + F(n-2)$ for $n \geq 2$, where F is the Fibonacci sequence with $F(0) = F(1) = 1$.

Problem 8. Create a machine that will recognize strings representing legitimate decimal numbers in the following form: at least one of the digits 0-9, possibly preceded by a hyphen (-), and possibly separated by a single decimal point (the decimal point can come at the beginning or the end).

1. (2 pts) Write a regular expression to represent the legitimate numbers.

2. (8 pts) Thinking of the set of inputs as $\{d, -, .\}$, where d is any digit, create a machine that will recognize only legitimate numbers.

Problem 9. (20 points total) A retailer ("Wacky Andy's") sells four "tech" items (plus cold beer): computers (x_1), cameras (x_2), printers (x_3), monitors (x_4). Andy's wacky (admittedly clumsy and silly!) ad campaign: "Buy a computer, get a free jump drive! Buy a computer and printer, get a free jump drive! Buy a computer and monitor, get a free jump drive! Buy a computer, camera, and printer, get a free jump drive! Buy a camera, get a free jump drive! Buy a printer and monitor, get a free jump drive! Buy a printer, get a free jump drive! Buy absolutely nothing, get a free jump drive! Any other combinations: sorry, no jump drive for you!"

1. Fill in the truth table below, where a "1" indicates that a free jump drive is included in the shipment:

x_1	x_2	x_3	x_4	$f(x_1, x_2, x_3, x_4)$
1	1	1	1	
1	1	1	0	
1	1	0	1	
1	1	0	0	
1	0	1	1	
1	0	1	0	
1	0	0	1	
1	0	0	0	
0	1	1	1	
0	1	1	0	
0	1	0	1	
0	1	0	0	
0	0	1	1	
0	0	1	0	
0	0	0	1	
0	0	0	0	

2. (8 pts) To the right of the table above, draw the corresponding Karnaugh map, and use it to find a minimal sum-of-products for the truth function.

3. (4 pts) Below, draw the minimized logic network to help your confused shipping department.

4. (8 pts) Problem 9, continued. Use the Quine-McCluskey algorithm to minimize the cannonical sum-of-products.

Problem 10. Consider the following machine, with states given as numbers 0-6:

Present State	Next State, Present Input 0	Next State, Present Input 1	Output
0	3	5	1
1	1	6	1
2	0	4	0
3	1	6	1
4	5	3	0
5	4	1	0
6	2	5	1

1. (8 pts) Draw the state graph for the machine.

2. (2 pts) Compute the output sequence for the input sequence below:

Time	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8
Input	0	1	0	1	0	1	1	0	
State	0								
Output									

Extra Credit (4 pts). Minimize the machine of problem 10.