

MAT385 Test 1, Spring 2006: Chapters 1 and 2

Name:

Directions:

- All problems are equally weighted.
- Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning).
- Indicate clearly your answer to each problem (e.g., put a box around it).
- **Good luck!**

Problem 1 (5pts each).

1. For the following wff find an interpretation in which it is true and one in which it is false:

$$(\exists x)[A(x) \wedge (\forall y)B(x, y)]$$

True	Domain	A	B
False	Domain	A	B

2. Prove the following: Every bear eats only fish, and some bear eats; therefore, there is a fish.
[Use $B(x)$, $E(x,y)$, $F(x)$.]

Problem 2. Demonstrate that

$$1 + r + r^2 + r^3 + \dots + r^n = \frac{1 - r^{n+1}}{1 - r}$$

1. (5pts) Directly

2. (5pts) By induction.

Problem 3. A recent song by Rilo Kiley inspired this one: we define the command “leads-to(x,y)” to mean that x leads to y. Here’s the Prolog database:

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leads-to(kissing,touching).
leads-to(touching,sex).
leads-to(marriage,sex).
leads-to(candy,kissing).
leads-to(dinner,kissing).
leads-to(liquor,kissing).
leads-to(love,marriage).
```

1. (2pts) Trace execution of the command *leads-to(X,kissing)*.
2. (4pts) Define the recursive command *doomed-to*, based on *leads-to*, which would be used to indicate that “once *X* happens then *Y* follows” using the syntax *doomed-to(X, Y)*.
3. (4pts) Trace execution of the command *doomed-to(X,sex)*.

Problem 4. Prove or disprove the following using propositional logic:

1. (5pts) If I am the Pope, then I have land in Florida to sell you. I have land in Florida to sell you. Therefore I am the Pope. [Use statement letters P(ope I am), L(and for sale).]

2. (5pts) If NKU goes NCAA Division 1, then football will follow. Basketball is the “marquee sport” at NKU. Football at NKU will mean basketball is no longer the marquee sport. Therefore, if NKU basketball is the marquee sport, then NKU has not gone Division 1. [Use statement letters D(ivision 1), (F)ootball at NKU, B(asketball marquee sport).]

Problem 5. Rose and Bud had one child, Spud. Spud put a curse on his family by instituting the following family tradition (to start with himself): each child in the family must have one more child than his or her parents.

1. (4pts) Let $P(n)$ be the “progeny function” of Rose and Bud, which gives the number of individuals in generation n (with Spud being generation $n = 1$). Write a linear recurrence relation for $P(n)$.
2. (4pts) In class we used the “expand, guess, and check” method to find the closed form solution. Use this procedure to find the closed form solution for $P(n)$.
3. (2pts) In which generation will the descendants of Rose and Bud outnumber the Earth’s current human population (US Census Bureau estimates it at 6.5 billion people)?

Problem 6. Let P denote the statement “Paul is rich” and let Q denote the statement “Paul is happy”.

1. (4pts) Write each of the following statements in the form of a wff:

(a) If Paul is rich, then he is unhappy.

(b) Paul is neither rich nor happy.

(c) Paul is not rich only if he is happy.

(d) Paul is rich if and only if he is not happy.

2. (6pts) For the interpretation in which the domain is all people, let $P(x)$ be the statement that “ x is rich”, and $Q(x)$ be the statement that “ x is happy”. Prove the following:

$$(\forall x)[P(x) \rightarrow Q(x)] \wedge (\exists x)Q(x) \longrightarrow (\exists x)P(x)'$$

Problem 7. True or False?

1. () All the rules of propositional logic are retained in predicate logic.
2. () BinarySearch is order $\log_2(n)$ on a sorted list, while sequential search is order n .
3. () One underutilized proof technique is “proof by converse”, which means proving $Q' \longrightarrow P'$ rather than $P \longrightarrow Q$.
4. () The algorithm MergeSort requires 12 comparisons for the lists $\{4, 20\}$ and $\{1, 2, 5, 8, 7, 9, 11, 13, 18, 19, 21\}$, demonstrating its “worst case” behaviour.

Extra Credit (2pts): Upon your death you find yourself before a two-headed monster guarding two tunnels: one leads to heaven, the other to hell. A sign informs you that one head always lies, while the other always tells the truth (although you don't know which is which); furthermore you may ask exactly one question (of either head) to help find your way to heaven.

What one question can you ask that will guarantee a trip to heaven (or hell, if you'd rather!)?