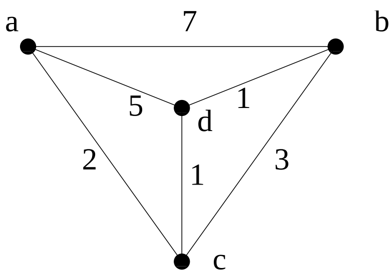
MAT385 Spring 2006, Test 2: Chapters 3.1, 5, 6, and 7.1-2

Name:

Directions:

- Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning).
- Indicate clearly your answer to each problem (e.g., put a box around it).
- **Good luck!**

Problem A (20 points). Given the graph below,



1. (2 pts) Fill in the adjacency matrix representing the graph (be careful: you'll need this!)

$$\begin{array}{c|cccc} & a & b & c & d \\ a & & & & \\ b & & & & \\ c & & & & \\ d & & & & \end{array}$$

2. (2 pts) Are there advantages to using an adjacency list (versus an adjacency matrix) here?

3. (4 pts) Use Dijkstra's algorithm to find the shortest distance from node a to b, and the path.

4. (4 pts) Use Bellman-Ford's algorithm to find the shortest distance from node a to every other node. Give the paths from a to every other node as well.

5. (4 pts) Use Floyd's algorithm to find the shortest distance between every pair of nodes.

6. (4 pts) How many distinctly different Hamiltonian circuits start from node a? What is the minimal weight for a circuit?

Problem B (10 points). Here's the situation: you've got to placate three difficult guests at your party. If they're not happy, then no one's happy. Your first task: to write the happiness function, $H(g_1, g_2, g_3)$. Here's how it works:

- If all three are in the same room, they're happy (because they can spy on each other!).
- If guest g_1 is out of the room and the other two are in, then they're all happy. Similarly, if g_1 is in while the other two are out, then they're all happy.
- If none are in the room, then they're happy.

1. (2 pts) Write the Happiness truth function for the situation. Let "1" signify being in the room.

g_1	g_2	g_3	$H(g_1, g_2, g_3)$
1	1	1	
1	1	0	
1	0	1	
1	0	0	
0	1	1	
0	1	0	
0	0	1	
0	0	0	

2. (2 pts) In the space to the right of the table above, write the canonical sum of products representations for the Happiness function.
3. (3 pts) Simplify the canonical sum of products representations for the Happiness function as best you can, using whatever Boolean algebra laws you can. How many AND, OR, or NOT gates will you need in your final simplified version?

4. (3 pts) Create a corresponding hardware implementation (logic network) that will be part of a system to monitor the room via video and give you an electric shock if your inputs suggest that your three difficult guests are <u>unhappy</u>.

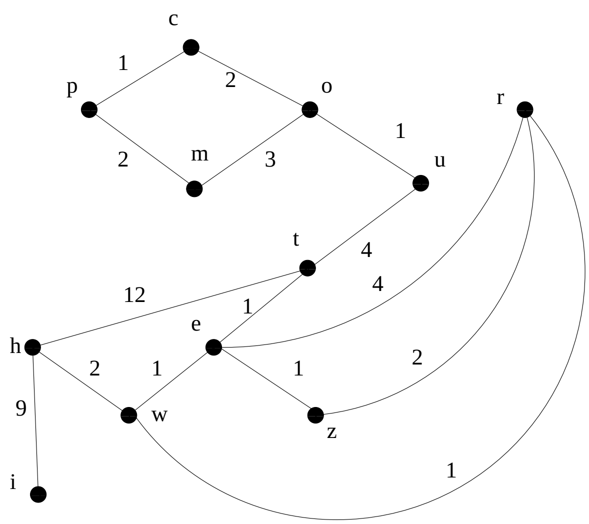
Problem C (10 points). Given the set of letters $\{m\ o\ i\ e\ t\ r\ c\ p\ h\ z\ u\ w\}$.

1. (4 pts) Populate a binary search tree using the data given in the order above. What's the depth of this tree?

2. (3 pts) Do an inorder and preorder traversal of the tree. What do you notice about the inorder traversal?

3. (3 pts) What's the minimal possible depth of a binary search tree constructed using this data? If less than the current depth, reorder the data so that it achieves its minimum, and draw the tree.

Problem D (10 points) Consider the graph given below, using the same data as problem C:



1. Do a depth-first traversal of the graph starting from node c.

2. Do a breadth-first traversal of the graph starting from node c.

3. Use Kruskal's algorithm to create a minimal spanning tree. Draw the tree (you may trace it over the tree above), and give its computed weight.

4. Is there either a Hamiltonian circuit or an Euler path for this graph? Explain.

Problem E (10 points). Given the set of letters $L = \{m\ o\ i\ e\ t\ r\ c\ p\ h\ z\ u\ w\}$ as in Problem D.

1. How many distinct elements are in the Cartesian product $L \times L$?

2. Decide if the following are true or false (mark each T or F):
   - $\{m\} \in L$
   - $\emptyset \in L$
   - $L \subseteq L$
   - $m, o, i \subset L$

3. When we create a binary search tree, we input the elements in a given order.
   - How many different orders are there for the elements of L?
   - Does each different order create a different binary search tree?

4. How many possible subsets are there of the letters in L? [Hint: this is the size of the power set of L].

Problem F (10 points). Prove the following for all Boolean algebras:

1.

$$x + x = x$$

2. (half-adder)

$$x_1' x_2 + x_1 x_2' = (x_1 + x_2) \cdot (x_1 \cdot x_2)'$$

3.

$$x + (x' \cdot y) = x + y$$

4.

$$x \cdot x = x$$