MAT385 Final (Fall 2007)

Name:

Directions:

a. All problems are equally weighted. You **must** skip one of problems 1-6, but you **may not** skip problems 7-10. Write "SKIP" clearly on the problem you wish to skip.

b. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning).

c. Indicate clearly your answer to each problem (e.g., put a box around it).

d. **Good luck!**

Problem 1. Prove the following two arguments:

a. If I were in Bermuda, I would be happy. I would not have to take this test if I were happy. Therefore, if I have to take this test, I am not in Bermuda. [Use statement letters B, H, T.]

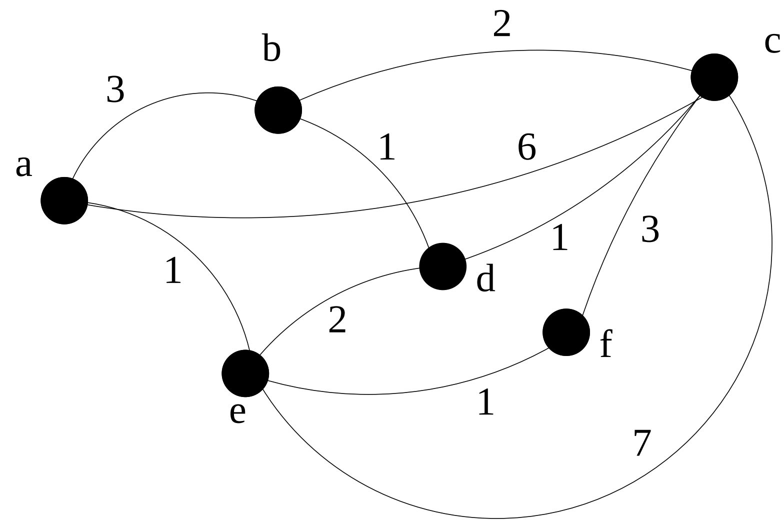
b. Every professor talks only to students, and some professor talks to someone. Therefore there is a student. [Use P(x), T(x,y), S(x).]

Problem 2. Prove **your choice (but only one)** of the following two statements:

a. The power set of a set of n elements contains 2^n elements, for $n \geq 0$.

b. The number of two element subsets of a set with $n \geq 2$ elements is $\frac{n(n-1)}{2}$.

Problem 3. Consider the following graph:



a. (6pts) Use Dijkstra's method to find the shortest path from node a to node c.

Step 1:		a	b	c	d	e	f
	d						
	s						

Step 2:		a	b	c	d	e	f
	d						
	s						

Step 3:		a	b	c	d	e	f
	d						
	s						

Step 4:		a	b	c	d	e	f
	d						
	s						

Step 5:		a	b	c	d	e	f
	d						
	s						

Step 6:		a	b	c	d	e	f
	d						
	s						

b. (1pt) Is this graph planar?

c. (1pt) Does this graph have an Euler path? If so, describe it.

d. (1pt) Is there a Hamiltonian circuit starting from node c? If so, describe it.

e. (1pt) Draw a minimal spanning tree for the graph (trace it on the graph).

Problem 4.

a. (2pts) Suppose that we create a binary search tree using the 26 letters of the alphabet. What is the minimal possible depth of the tree? (Show your reasoning!)

b. (4pts) Consider the list of the 26 letters in order, abcdefghijklm nopqrstuvwxyz, indexed from 1 to 26. Use the following "divide and conquer" algorithm to create a single binary tree from this list:

- If list size n is even, divide the list in half; set the root as element $n/2$; recurse on the "halves" remaining, attaching their roots to current root to form a binary tree.
- If list size n is odd, set the root as the middle element; recurse on the "halves" remaining, attaching their roots to current root to form a binary tree.

Single children are right children. Draw the tree.

c. (1pt) Is your tree a binary search tree?

d. (3pts) Write the following traversals of the tree:

i. preorder

ii. postorder

iii. inorder

Problem 5. Consider the recurrence relation $P(n+1) = 2P(n) + P(n-1)$, with $P(0) = 1$ and $P(1) = 3$.

a. (2pts) Fill in the following table:

$P(0)$	
$P(1)$	
$P(2)$	
$P(3)$	
$P(4)$	
$P(5)$	

b. (6pts) Demonstrate that $P(n) \geq 2^n$ for $n \geq 0$.

c. (2pts) Find a closed-form solution for $P(n)$. (You do not need to prove it!)

Problem 6. Consider an unweighted graph to represent a house, as follows:

a. (4pts) The rooms of a house (and the "Outside") are the nodes. The rooms (with "nicknames") and modified "adjacency lists" are as follows:

  i. K(itchen) $\rightarrow$ D $\rightarrow$ L $\rightarrow$ Base $\rightarrow$ O
  ii. L(iving) $\rightarrow$ D $\rightarrow$ K $\rightarrow$ Bed1 $\rightarrow$ Bed2 $\rightarrow$ O
  iii. D(ining) $\rightarrow$ L $\rightarrow$ K
  iv. Bed1 $\rightarrow$ L $\rightarrow$ Bath
  v. Bed2 $\rightarrow$ L $\rightarrow$ Bath
  vi. Bath(room) $\rightarrow$ Bed1 $\rightarrow$ Bed2
  vii. U(tility) $\rightarrow$ Base
  viii. Base(ment) $\rightarrow$ U $\rightarrow$ K $\rightarrow$ O
  ix. O(utside) $\rightarrow$ Base $\rightarrow$ K $\rightarrow$ L

  Above right, draw a planar graph that represents the house.

b. (2pt) If rooms are separated by doors, how few doors might the house contain?

c. (2pt) List the rooms by making a breadth-first traversal of the graph from the node Bed1.

d. (2pt) List the rooms by making a depth-first traversal of the graph from the node Bed1.

Problem 7.

a. (4pts) Draw the logic network given by the Boolean expression $(y' \cdot x) + x + (y + x) \cdot y'$

b. (4pts) Use properties of Boolean algebra to demonstrate that the logic network can be replaced by an equivalent network represented by the expression $x + (y' \cdot x)$.

c. (2pt) Can you simplify this logic network even further?

Problem 8. Consider the following truth function:

x_1	x_2	x_3	x_4	$f(x_1, x_2, x_3, x_4)$
1	1	1	1	0
1	1	1	0	0
1	1	0	1	0
1	1	0	0	1
1	0	1	1	1
1	0	1	0	1
1	0	0	1	0
1	0	0	0	1
0	1	1	1	0
0	1	1	0	0
0	1	0	1	0
0	1	0	0	0
0	0	1	1	1
0	0	1	0	1
0	0	0	1	0
0	0	0	0	1

a. In the space above and to the right, minimize the truth function using the Karnaugh map.

b. Ditto below, using Quine-McCluskey.

Problem 9. Consider the regular expression $(0 \vee 1)0*1*(101)*$.

a. (2pts) Determine if the strings following are in the regular set determined by the regular expression:

   i. λ

   ii. 0101101

   iii. 101101101101

   iv. 0000000000

b. (8pts) Construct a finite state machine to recognize the regular expression.

Problem 10. For the finite state machine given by the state table below:

a. (2 pts) Compute the output sequence for the following input sequence:

Time	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7
Input	0	1	0	1	1	0	1	
State	0							
Output	0							

Present State	Next State		Output
	0	1	
0	1	2	0
1	3	4	1
2	3	4	1
3	1	4	1
4	5	4	1
5	8	6	0
6	8	7	1
7	5	8	0
8	8	8	0
9	0	3	0

b. (8pts) Minimize the machine, and draw a state graph corresponding to the minimized machine.