MAT385 Test 1 (Fall 2007): Logic, Proofs, Recursion

Name:

Directions: All problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. Prove the following two arguments:

a. If my uncle is not hungry, then my aunt is not sick. If my uncle is hungry, he's an ogre. Therefore, if my uncle is in good spirits (i.e. not an ogre), my aunt is feeling fine. (H, S, O)

   i. (2 pts) The propositional wff to prove:

   ii. (3 pts) The proof:

b. (5pts) If the war was justified, then there were WMD. There were no WMD. 1=2. Therefore the war was justified.

   Note: you may use any ordinary number facts.

Problem 2. Prove that

$$(A \to B) \to A' \vee B$$

is a tautology by two methods:

a. (5pts) by truth tables,

A	B				$(A \to B) \to A' \vee B$
T	T				
T	F				
F	T				
F	F				

b. (5pts) and by algorithm Tautology Test (you may **not** use the "implication" equivalence rule, of course, but you may use other basic rules such as De Morgan and simplification).

Problem 3. If something is slithy, then it is uffish. Therefore, if there is a slithy something, then there is an uffish something.

a. (2 pts) Write this argument as a predicate wff, using “S” for slithy and “U” for uffish.

b. (8 pts) Either prove that the wff is a valid argument or give an interpretation in which it is false.

Problem 4. Prove that the product of an irrational number and a rational number (not equal to zero) is irrational.

Notes:

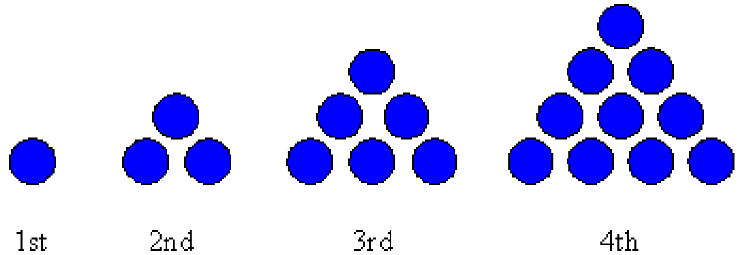
a. Rational numbers are of the form: p/q, where p and q are integers, $q \neq 0$; irrational numbers aren't of that form!

b. If you're struck, I'll sell you a suggestion for two points.

Problem 5. Prove that $n^2 \geq 2n - 1$ for $n \geq 1$.

Prove it in a completely different way for 4 points of extra credit.

Problem 6. Early members of the Pythagorean Society defined *figurate numbers* to be the number of dots in certain geometrical configurations. The first few *triangular numbers* are 1, 3, 6, and 10:



Riwa's Triangular Numbers

a. (2 pts) Compute the next four triangular numbers.

b. (8 pts) Find and solve a recurrence relation for the closed form solution of the n^{th} triangular number.

Problem 7. One strategy for computing a^n is to multiply a by itself $n-1$ times:

$$a^n = a * a * \ldots * a \tag{1}$$

Consider another algorithm, *pow(a,n)*. For simplicity of comparison, assume that $n = 2^m$, where m is a positive integer. Hence, we could define *pow(a,n)* as follows:

function pow(a,n):
if n=1, return a; else return square(pow(a,n/2))

a. (5 pts) Write the computation of $pow(a, n)$ as a "divide and conquer" recursion equation, where we count only the number of multiplications.

b. (4 pts) Find a closed form solution for the number of multiplications for $pow(a, n)$.

c. (1 pt) Describe precisely how $pow(a, n)$ compares with the straight-ahead algorithm in (1).