MAT385 Test 1 (Spring 2008): Logic, Proofs, Recursion

Name:

Directions: All problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1.

a. Prove the following argument:

   If Albert is happy, then Bert is sad. Either Bert is sad or Cathy is angry. But Bert is not sad. Therefore Cathy is angry, and Albert is unhappy. (A, B, C)

b. Write (and distinguish) these two arguments using predicate logic (use HS(x) and G(x)):

   i. God helps those who help themselves.

   ii. God only helps those who help themselves.

Problem 2. Consider the following Prolog database, where by "student(x,y)" we mean that x is a student of y:

```
student(beth, andy)
student(jen, andy)
student(andy, don)
student(tailiang, don)
student(don, pierce)
student(cliff, pierce)
student(andy,cliff)
```

a. Write a recursive definition of binary Prolog rule "educated", where a person (e.g. Bob) has been educated by someone else (say Mary), written

   educated(bob,mary)

   if Bob were a student of Mary, or if Bob were a student of someone else educated by Mary.

b. Trace execution of the command **educated(beth,X)**

c. Trace execution of the command **educated(X,don)**

Problem 3. Prove the following argument for all $n \geq 1$

$$1 + 3 + 5 + ... + (2n - 1) = n^2$$

a. by induction

b. and directly, using the result

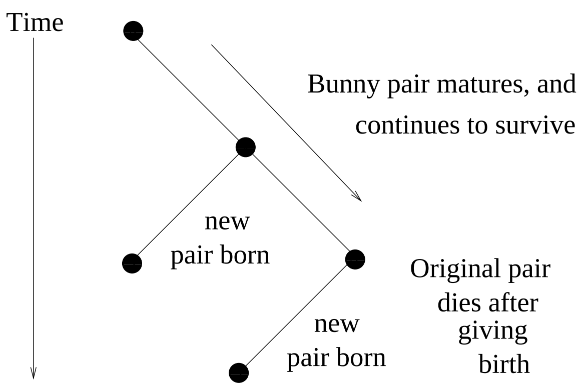
$$1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$$

Problem 4. Prove the following argument of Predicate Logic:

$$(\forall x)(A(x) \to B'(x)) \wedge (\exists x)(B(x) \wedge C(x)) \to ((\forall x)A(x))'$$

Problem 5. Demonstrate that the square of an odd integer is equal to $8k + 1$ for some integer k.

Problem 6. In class I discussed a variation of the Fibonacci numbers, within the context of rabbits. Fibonacci assumed immortal rabbits; suppose we assume mortal rabbits, that die at the end of their third month (so a pair appears in only three months' censuses). At the end of each of months two and three, a pair produces a new pair of bunnies (which live the same life cycle as their parents). The lifecycle can be represented as follows:



a. Find the number of pairs P_i for the first five generations (where the first pair is generation one).

P_1	
P_2	
P_3	
P_4	
P_5	

b. Determine the recurrence relation that this system satisfies.

Problem 7. Solve the following recurrence relation subject to the basis step:

$$T(1) = 3$$
$$T(n) = T(n/2) + n$$

Make sure that you check your result using several values of n.