MAT385 Test 2 (Spring 2008): Sets, Graphs, Trees

Name:

Directions: Problems are **not equally weighted**. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1 (10 pts) . Consider the following arithmetic expression in infix notation:
((3*8)-5)+(11-(7/5))

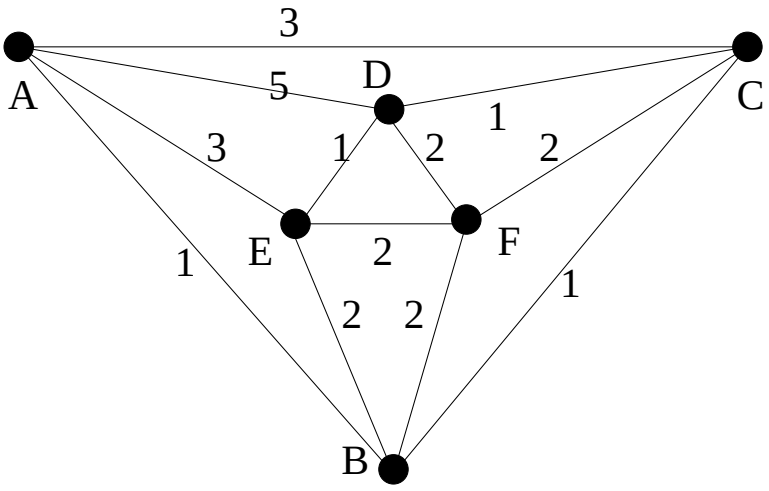
a. (2 pts) Draw the corresponding expression tree.

b. (4 pts) Give the corresponding preorder and postorder traversals of the expression tree.

c. (4 pts) Given the following traversals, construct a legitimate corresponding expression tree:

$$\begin{array}{rl} preorder: & + \; - \; 5 \; 4 \; * \; / \; 6 \; - \; 2 \; 3 \; 9 \\ postorder: & 5 \; 4 \; - \; 6 \; 2 \; 3 \; - \; / \; 9 \; * \; + \end{array}$$

Problem 2 (20 pts) The following graph is associated with the octahedron, one of the platonic solids. In the space to the right, write down the adjacency matrix (1 pt).



a. (2 pts) Was I able to trace the graph above without lifting my pencil from the paper, and without duplicating any edges? Why or why not?

b. (8 pts) Perform Dijkstra's algorithm on the graph, starting at the node A and going to the node D. Give the initial values of d and s, and then their values at termination:

	A	B	C	D	E	F
d						
s						

	A	B	C	D	E	F
d						
s						

	A	B	C	D	E	F
d						
s						

c. (8 pts) Use Bellman-Ford starting from node D, giving the initial values of d and s and then their values after the first two iterations:

	A	B	C	D	E	F
d						
s						

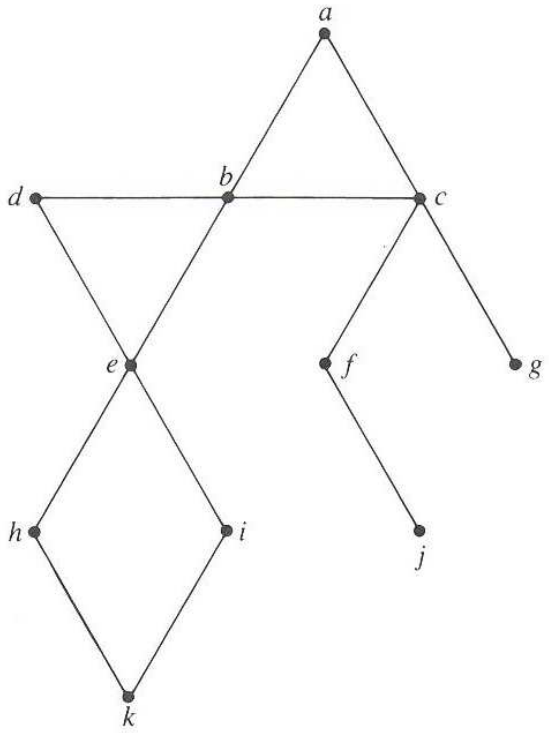
	A	B	C	D	E	F
d						
s						

	A	B	C	D	E	F
d						
s						

d. (2 pts) Give an adjacency list representation of the graph.

Problem 3 (10 pts) .

a. (5 pts) Do a breadth-first traversal of the following graph starting from node h:



b. (5 pts) Now do a depth-first traversal from node h.

Problem 4 (10 pts) . Consider the set S of 12 students in a discrete math class.

a. (2 pts) How many subsets of S are there?

b. (4 pts) Use a tree and symmetry to calculate the number of ways in which a trio of students can be chosen to form a team of three students. Suppose that the order is unimportant for team members.

c. (4 pts) Consider two (arbitrary) subsets A and B of S. Draw a Venn diagram for this situation, in the most general case, and find the number of **distinct sets** that one can construct inside the Venn diagram using only the operations of union, intersection, and complements. Justify!

Problem 5 (10 pts) .

a. (5 pts) Draw all possible (non-isomorphic) bipartite-complete graphs on eight nodes, including trivial ones, and give them labels A, B, ..., etc. in some sensible way.

b. (1 pts) Which ones are trees, or could be construed as trees?

c. (2 pts) Which ones admit Hamiltonian circuits?

d. (2 pts) Which ones admit Euler paths?

Problem 6 (10 pts) .

a. (5 pts) Demonstrate that any binary tree of depth d has at most $2^{d+1} - 1$ nodes.

b. (5 pts) Given a sorted list of 16 distinct elements. In searching the list for an element **known to be in the list**, how many comparisons must one make in the worst case using binary search?