

# MAT385 Test 1 (Spring 2009): Logic and Proofs

Name:

**Directions:** Problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

**Problem 1.** (10 pts)

a. Construct a truth table for the following wff:  $(A \rightarrow B) \rightarrow [(A \vee B) \rightarrow (B \vee C)]$

$A$	$B$	$C$
$T$	$T$	$T$
$T$	$T$	$F$
$T$	$F$	$T$
$T$	$F$	$F$
$F$	$T$	$T$
$F$	$T$	$F$
$F$	$F$	$T$
$F$	$F$	$F$

b. Prove the wff using propositional logic.

**Problem 2.** (10 pts) The following is one of Lewis Carroll's puzzles. Assume each statement is true (and use I - insane, L - logic can do, J - Jury service, S - Son of you as statement letters):

1. Every one who is sane can do logic.
2. No lunatics are fit to serve on a jury.
3. None of your sons can do logic.
  - a. Use predicate logic to show that no son of yours would serve on a jury.

b. Write the converse of each statement in English (are any of these true?).

- 1.
- 2.
- 3.

c. Write the contrapositive of each statement in English (are any of these true?).

- 1.
- 2.
- 3.

**Problem 3.** (10 pts)

a. Prove the following:

It's Abe Lincoln's birthday only if it's February.  
Lincoln's birthday is a necessary condition for Charles Darwin's birthday.  
Therefore if it's not February, then it's not Darwin's birthday.

b. Consider the following wff:

$$(\forall x)[P(x) \rightarrow Q(x)] \rightarrow [(\exists x)P(x) \rightarrow (\forall x)Q(x)]$$

Determine whether the wff is valid or not. If it is valid, prove it; otherwise, give examples of interpretation in which it's true and in which it's false.

**Problem 4.** (10 pts)

a. Prove that if  $x = 2$  and  $x = 5$ , then the moon is made of green cheese.

b. Either prove or give an interpretation in which the following is false:

$$(\forall x)P(x) \vee (\exists x)Q(x) \rightarrow (\forall x)[P(x) \vee Q(x)]$$

**Problem 5.** (10 pts) The sum of even integers is even.

a. Do a direct proof.

b. Do a proof by contraposition.

c. “The sum of odd integers is odd.” Or is it? If it’s true, prove it; if it’s false, modify it to say something **interesting** (and true)!