MAT385 Final (Fall 2010): Boolean Algebras, FSM, and old stuff

Name:

Directions: Problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

You may skip one of the "Old Stuff" problems (problem 1-7), and even one of the "New Stuff" problems (8-12). Write "skip" clearly on the ones you don't want me to grade.

Problem 1.

- Jesus: (Luke 16:15): And he said unto them, "Ye are they which justify yourselves before men; but God knoweth your hearts: for **that which is highly esteemed among men is abomination in the sight of God**." (my emphasis)
- Andy: Mathematics is highly esteemed among men.

a. (8 pts) Given the statements of these two men and predicate logic, prove that there is something that is an abomination in the sight of God.

b. (2 pts) Write the negation of Jesus's (emphasized) proposition:

Problem 2. Use logic to prove that among all animals, some egg-laying animal is not a snake, given that

a. All birds lay eggs.

b. Some animals that lay eggs are wary of snakes.

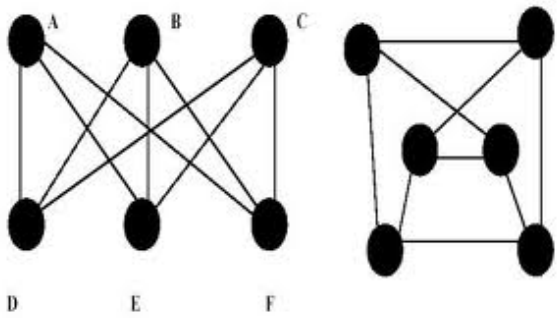
c. Snakes are wary of snakes.

d. Not all birds are wary of snakes.

(use $B(x), L(x), W(x), S(x)$).

Problem 3. Graphs:

a. (4 pts) Determine whether the following graphs are isomorphic or not:



b. (2 pts) Determine whether the graph at left is planar, and, if so, verify Euler's formula for planar graphs.

c. (4 pts) For the graph at left, determine whether an Euler path exists and whether a Hamiltonian circuit exists (represent them graphically, if they exist).

Problem 4. Recursion:

a. (3 pts) Draw all distinct simple graphs of 1, 2, and 3 vertices.

b. (4 pts) Now: describe a recursive scheme for drawing all simple graphs of $n+1$ vertices given all simple graphs of n vertices (redundant representations are okay – we just want to make sure that we have them all).

c. (3 pts) Illustrate your scheme on the simple graphs of 4 vertices.

Problem 5. Consider the Fibonacci numbers, given by $F(1) = 1$, $F(2) = 1$, and $F(n+1) = F(n) + F(n-1)$: demonstrate that, for $n \geq 2$,

$$F(n)F(n-1) = \sum_{i=1}^{n-1} F(i)^2$$

Problem 6. (6 pts) Consider the calculation $[(6 \div 2) * 4] + [(1 + x) * (5 + 3)]$.

a. Draw its expression tree.

b. Give the following traversals of this tree:

i. preorder

ii. postorder

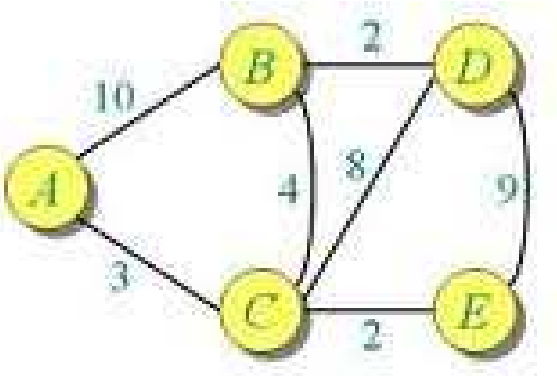
(4 pts) Draw a tree whose inorder traversal is

$f, a, g, b, h, d, i, c, j, e$

and whose postorder traversal is

$f, g, a, h, i, d, j, e, c, b.$

Problem 7. Consider the following graph:



a. (3 pts) Write down the adjacency matrix. Explain why this is not the most efficient method for storing the graph.

b. (7 pts) Carry out Dijkstra's algorithm to find the shortest route from node A to node D.

Problem 8. Recall problem 3 on the 7.1 homework: Let S be the set $\{0, 1\}$. Then S^2 is the set of all ordered pairs of 0s and 1s; $S^2 = \{(0,0), (1,0), (0,1), (1,1)\}$. Consider the set B of all functions mapping S^2 to S. For example, one such function $f(x, y)$ is given by

$$f(0,0) = 0$$
$$f(0,1) = 1$$
$$f(1,0) = 1$$
$$f(1,1) = 1$$

a. (2 pts) How many elements are there in B? (justify)

b. (2 pts) How many **commutative** functions are there in B? (justify)

c. (6 pts) Given that we define "+" by $(f_1 + f_2)(x, y) = max(f_1(x, y), f_2(x, y))$; define appropriate elements of B and operations on B so that $[B, +, \cdot, ', 0, 1]$ is a Boolean algebra.

Problem 9. Recall how we used a syllogism of Lewis Carroll to create a finite state machine, guided by a quote from T. H. White's *The Once and Future King*: on a sign above an ant colony are the words "Everything not forbidden is compulsory". Let's take that as our guiding principle, and assume that

- All birds lay eggs.
- Some animals that lay eggs are wary of snakes.

a. (4 pts) Write the truth table for the three statement letters B, L, W (Birds, Lay, Wary) and the truth function $compulsory(B, L, W)$.

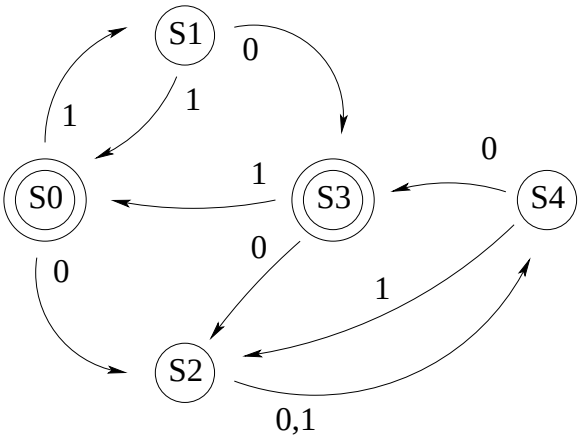
b. (3 pts) Draw the Karnaugh map for the truth function $compulsory(B, L, W)$.

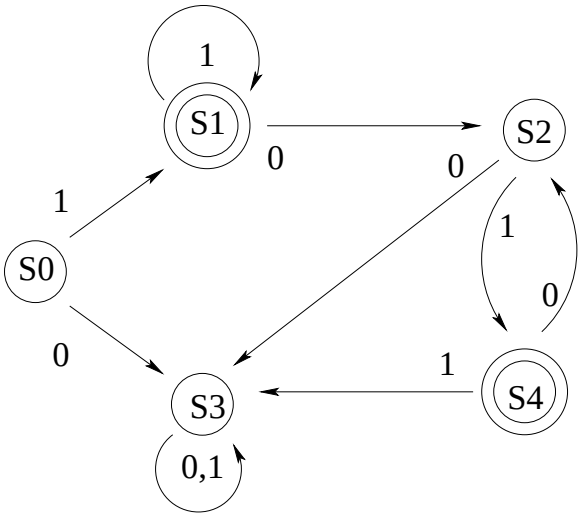
c. (3 pts) Write a minimal sum-of-products expression for $compulsory(B, L, W)$.

Problem 10. Use Quine-McCluskey to find a minimal sum-of-products form for the following truth function: [If you can't do it by Quine-McCluskey, you'll get 5 points for doing it by Karnaugh.]

x_1	x_2	x_3	$f(x_1, x_2, x_3)$
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	1

Problem 11. Examine the follow Finite State Machines, which were built to recognize certain regular sets:





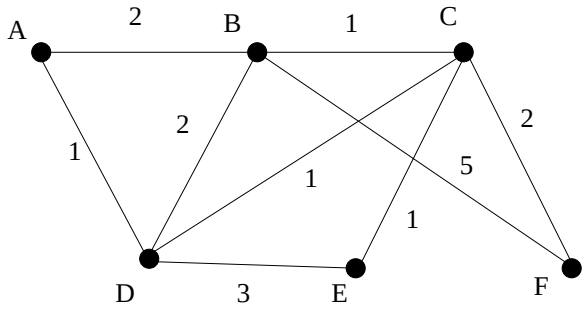
a. (3 pts) Determine the output sequence of the machine at left for the input 00100110.

Time	T1	T2	T3	T4	T5	T6	T7	T8	T9
	$S0$								

b. (3 pts) Determine what strings are recognized by the machine at right. You might start by writing down as many unique types of strings recognized by the machine as you can.

c. (5 pts) Extra credit – write a regular expression for the strings recognized by the machine at left.

Problem 12: Consider the following graph G:



a. Give a depth-first traversal, starting from node C.

b. Give a breadth-first traversal, starting from node C.