MAT385 Test 1 (Fall 2010): Logic and Proofs

Name:

Directions: Problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1: (10 pts)

a. Construct a truth table for the following wff: $[(A \vee B) \wedge C'] \rightarrow A' \vee C$.

b. Either prove or disprove the theorem given by the wff using propositional logic.

Problem 2: (10 pts) The following is one of Lewis Carroll's puzzles (in the domain of creatures; use G(x) - Greyhound, F(x) - Fat, R(x) - Runs well):

- No fat creatures run well;
- Some greyhounds run well.
- Therefore some greyhounds are not fat.

a. Write this syllogism as a theorem in predicate logic.

b. Use predicate logic to determine its truth value.

Problem 3: (10 pts) Prove, by any method: $F(n) = 5F(n-4) + 3F(n-5)$ for $n \geq 6$.

Problem 4: (10 pts) Consider the following wff:

$$(\forall x)[P(x)]' \rightarrow (\forall x)[P(x) \rightarrow Q(x)]$$

Determine whether the wff is valid or not. If it is valid, prove it; otherwise, give an example of an interpretation in which it's true and one in which it's false.

Problem 5: (10 pts) Given the set of propositions and the usual logical connectives, write a complete set of rules that define all well-formed formulas (wffs) **recursively**.

Problem 6: An obscure tribe has only three words in its language: moon, noon, and soon. New words are composed by juxtaposing these words in any order, as in soonnoonmoonnoon. Any such juxtaposition is a legal word.

a. Use the first principle of mathematical induction (on the number of subwords in the word) to prove that any word in this language has an even number of o's.

b. Use the second principle of mathematical induction (on the number of subwords in the word) to prove that any word in this language has an even number of o's.

Problem 7: True or False (with reasons)? [They might be tricky!]

a. [( )] If $0 = 1$, then pigs can fly.

b. [( )] Prolog proofs are really just examples of proofs by exhaustion.

c. [( )] As a general rule, existential generalization accompanies "$\wedge$" and universal generalization accompanies "$\rightarrow$".

d. [( )] The Factoral function values can be produced via a first-order, constant coefficient, linear recurrence relation.

e. [( )] The aim of a proof by contradiction is to produce a counter-example.