MAT385 Test 2 (Fall 2010): Sets, Graphs, Trees

Name:

Directions: Problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

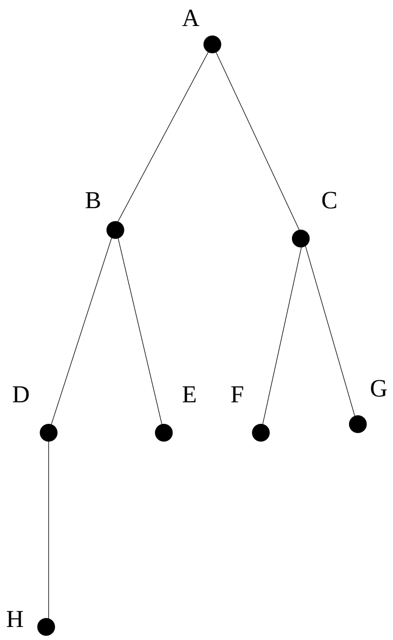
Problem 1: (10 pts) Consider the set of all natural numbers, $S = \{1, 2, 3, \ldots\}$.

a. (4 pts) Demonstrate that there are as many even natural numbers as there are all natural numbers: how is this possible?

b. (4 pts) Consider the proposed binary operation on S, given as follows: the image of two natural numbers is the sum of their squares. Show that this is indeed, a binary operation.

c. (2 pts) Give four examples of elements of the power set of S. Is $\{\emptyset\}$ an element of the power set?

Problem 2: (10 pts) Consider the following binary tree:



a. To the right of the tree, give the following traversals:

   i. pre-order

   ii. in-order

   iii. post-order

b. Draw the expression tree for the infix expression $(2 - z) * (7 - (x + y) * 2)$.

c. The postorder list of nodes is
   i, d, e, f, b, j, k, g, h, c, a.
   The inorder list of nodes is
   i, d, b, e, f, a, j, g, k, c, h.
   Draw the tree.

Problem 3: (10 pts) For a set of nine data items

- (2 pts) what is the minimum worst case number of comparisons a search algorithm must perform? [Give an integer!]

- (4 pts) Given $\{4, 7, 8, 10, 12, 15, 18, 19, 21\}$, find an order in which to enter the data so that the binary search tree has the minimum depth, and draw the search tree.

- (2 pts) Find the average number of comparisons done to search for an item that is known to be in the list using binary tree search on your tree given above.

Problem 4: (10 pts) Given the following two graphs, defined by their adjacency matrices (and where each entry represents the number of arcs between two vertices):

$$A = \begin{bmatrix} 0 & 1 & 2 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 2 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 1 & 2 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 0 & 2 & 1 \\ 1 & 0 & 2 & 0 & 1 \\ 0 & 2 & 0 & 1 & 1 \\ 2 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

a. (6 pts) Determine whether they are isormorphic or not.

b. (4 pts) List at least 8 criteria by which one can determine that two graphs are **not** isomorphic.

Problem 5: (10 pts) Consider the graph given by the following matrix: $A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$

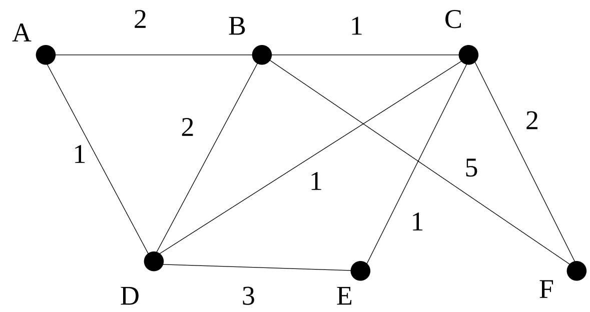
- Draw the graph, as elegantly as possible.

- Does the graph admit an Euler path? (justify!)

- Does the graph admit a Hamiltonian Circuit? (justify!)

- Is the graph planar? (justify!)

Problem 6: (10 pts) Consider the following graph G:



- (7 pts) Use the Bellman-Ford Algorithm to find the shortest distances between vertex A and all points. [If you would prefer, you can do Dijkstra's from A to F, but you'll only get 4 pts!]

- (3 pts) Draw a Minimal spanning tree for G, and give the minimal weight. What algorithm did you use?