MAT385 Final, Fall 2011

Name:

Directions:

- All problems are equally weighted. You may skip one problem: write skip on it.
- Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning).
- Indicate clearly your answer to each problem (e.g., put a box around it).
- **Good luck!**

Problem 1.

- Use propositional logic to prove the following argument valid:

  Babies are illogical. Illogical persons are despised. Nobody is despised who can manage a crocodile. Therefore babies cannot manage crocodiles. (B,I,D,M)

- Negate the preceding argument, to create as clean a wff as possible (don't just put a negation on it all!;).

Problem 2. Consider the following argument:

$$(\forall x)(\forall y)(T(x) \wedge H(y) \rightarrow L(x,y)) \wedge (\exists x)M(x) \wedge (\forall x)(M(x) \rightarrow H(x)) \rightarrow (\exists y)(\forall x)(T(x) \rightarrow L(x,y))$$

1. (3 pts) If the symbols T, H, L, M represent Teachers, Hard workers, Love (x loves y), and Math major, write the theorem in "plain English".

2. (7 pts) Prove the theorem.

Problem 3.

1. (5 pts) Describe all simple graphs with five vertices for which both Euler paths and Hamiltonian circuits exist. Are any non-planar?

2. (5 pts) How many non-isomorphic simple graphs with 4 nodes are there? Draw them.

Problem 4. Someone has poisoned the drink of exactly one of nine people in a room. We will find out which drink has been poisoned (and hence which of the nine people was the intended victim) using lab rats, but we want to harm as few as possible. Any mixture of the nine drinks will kill the lab rat (or get it drunk, another form of harm). Only a small quantity of the liquid is needed to kill one.

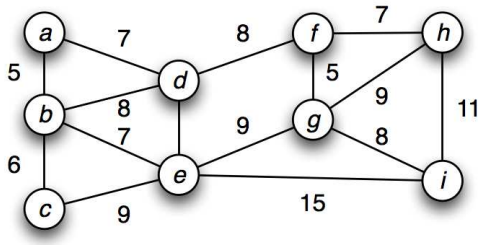
Because we're humane investigators, we want to design a sampling scheme to find the poisoned drink which has the optimal "worst case behavior" for harming rats. So we want to reduce the number of **tests** (but not necessarily deaths).

1. (2 pts) What is the number of leaves in the decision tree?

2. (3 pts) Find a lower bound on the number of tests required to find the poisoned drink in the worst case.

3. (5 pts) Create a decision tree to solve our problem that meets the lower bound in the worst case.

Problem 5. Use the following graph (note: add weight of 4 to the arc joining d and e) to carry out the following:



1. (5 pts) Use Dijkstra's algorithm to find the shortest path between nodes a and g.

2. (5 pts) Create the minimal spanning tree, using Kruskal's algorithm. Treating node f as root, traverse the tree

   (a) inorder

   (b) preorder

   (c) postorder

Problem 6. Solve the following recurrence relation subject to the given basis step:

$P(1) = 2$

$P(n) = 3(n+1)P(n-1) \quad n \geq 2$

Problem 7. Consider the following truth function:

x_1	x_2	x_3	x_4	$f(x_1, x_2, x_3, x_4)$
1	1	1	1	0
1	1	1	0	0
1	1	0	1	1
1	1	0	0	1
1	0	1	1	0
1	0	1	0	1
1	0	0	1	1
1	0	0	0	1
0	1	1	1	0
0	1	1	0	1
0	1	0	1	1
0	1	0	0	1
0	0	1	1	0
0	0	1	0	1
0	0	0	1	1
0	0	0	0	1

1. (5 pts) At right of the table, use the Karnough map to minimize the boolean expression.
2. (5 pts) Below, use Quine-McKluskey to minimize the boolean expression.

Problem 8. Write regular expressions for the following regular sets over the alphabet $\{0, 1\}$:

1. (2.5 pts) Finite strings consisting of only pairs of consecutive ones and/or consecutive zeros (including no pairs).

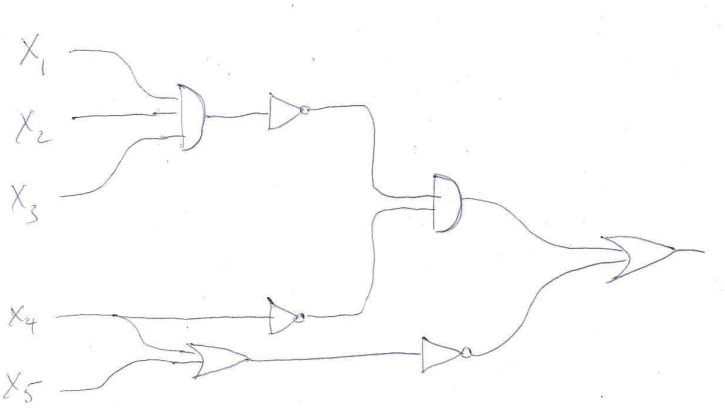
2. (2.5 pts) Finite strings containing odd numbers of pairs of "01".

3. (5 pts) Now build two machines, each of which recognizes one of those sets of strings.

Problem 9. Minimize the finite state machine given by the following state table, and draw the state graph of the minimized machine to the right of the table:

Present State	Next state 0	1	output
0	2	0	0
1	1	6	0
2	2	3	0
3	4	3	1
4	4	5	1
5	2	5	0
6	6	1	1

Problem 10. Consider the following (unoptimized) logic network:



1. (2.5 pts) Write the boolean expression for this network as drawn.

2. (2.5 pts) To the right of the network, draw an expression tree which will evaluate to this expression. Use vertices of $\cdot$, $+$, and $'$. Single children are left children.

3. (2.5 pts) Optimize the logic network, using whatever tools you'd like, writing it ultimately as a sum of products.

4. (2.5 pts) Draw the simplified logic network.