MAT385 Final, Spring 2013

Name:

Directions:

- All problems are equally weighted.
- The first part of the test is new material. You should skip one of those problems. The second part of the test is old material. You should skip one of those problems as well.
- Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning).

Good luck!

1 Part I: New Stuff

You must skip one of the following problems in Part I. Write "skip" clearly on it.

Problem 1. Prove the following, using and citing properties of Boolean Algebras:

$$x \cdot (z + y) + (x' + y)' = x$$

Problem 2. Use both the Karnaugh map and Quine-McCluskey to minimize the Boolean expression $x_1x_2x_3 + x_1'x_2x_3' + x_1x_2x_3' + x_1x_2'x_3$.

Here's a convenient table for you to use for the Karnaugh map:

And here's a starting table for Quine-McCluskey:

# of 1s	x_1 x_2 x_3

Problem 3. Here's the truth table we used to create the output of the full-adder (carry digit c and write digit s):

c_{i-1}	x_i	y_i	c	s
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Write a Boolean expression for each of the truth functions c and s (as functions of c_{i-1}, x_i, and y_i); then create a corresponding logic network for each. You may simplify if you wish.

Problem 4.

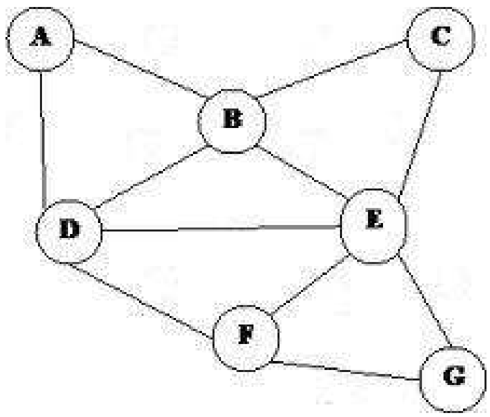
a. Design a machine to recognize the regular expression 1*0*1.

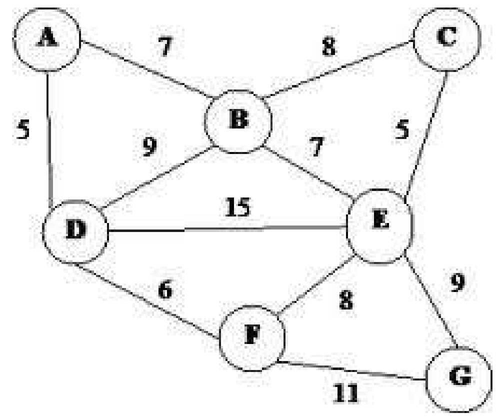
b. Minimize the following FSM (the sloppy delay FSM), and draw the minimized machine:

present	*next state*		
state	0	1	*output*
s_0	s_1	s_2	0
s_1	s_3	s_4	0
s_2	s_5	s_6	0
s_3	s_3	s_4	0
s_4	s_5	s_6	0
s_5	s_3	s_4	1
s_6	s_5	s_6	1

Then demonstrate the operation of the simplified machine with the input 01011 (followed by **either** a 0 or 1, to force out the last digit).

Problem 5. Consider the following two graphs:





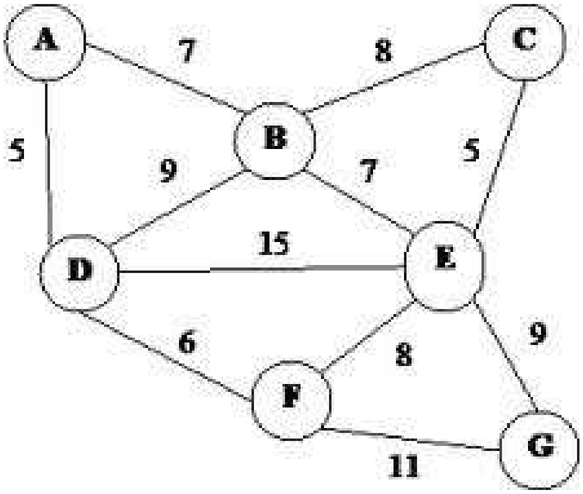
a. Carry out a traversal of the unweighted graph at left in each of the two ways, and record your answers below (while carrying out your work below):

   i. Depth-first from A: ______________________________

   ii. Breadth-first from E: ______________________________

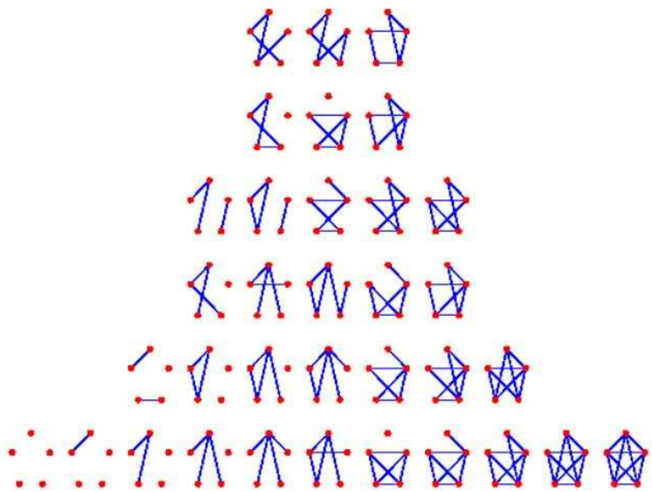
b. Find and draw a minimal spanning tree (and its weight) for the weighted graph at right. Give the minimal weight here: ______.

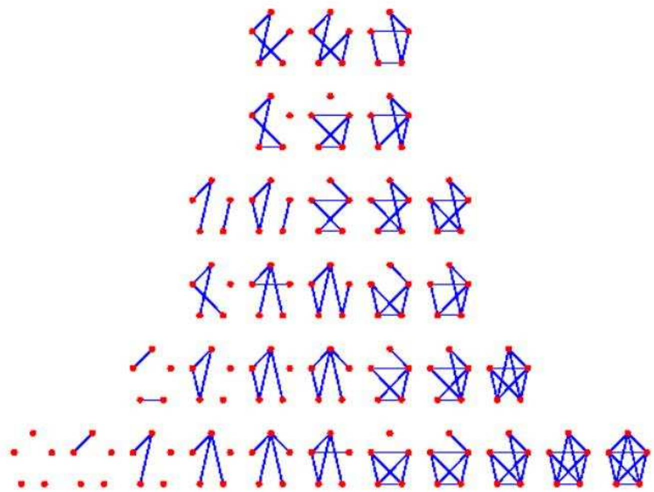
Problem 6. Illustrate Dijsktra's algorithm in finding the shortest path and associated cost in going



from A to G. Explain your steps, as though to teach the algorithm.

Problem 7. Consider all simple graphs on five vertices:





a. At left circle all those which possess an Euler path. Describe your strategy for deciding which have an Euler path. Alternatively cross out those that don't.

b. At right circle all those which have a Hamiltonian circuit (or cross out those that don't). Describe your strategy for deciding which have a Hamiltonian circuit.

Problem 8.

a. Construct the binary search tree for the data comprised of the words in this sentence, from left to right.

b. What's the depth of the binary search tree created from these words in

i. the worst case? ________

ii. in the best case? ________

How are these two extreme cases achieved?

c. What would be the maximum number of comparisons **necessary** for a spellchecker on a 100,000 word language?

2 Part II: Old Stuff

You must skip one of the following problems, too. Write "skip" clearly on it.

Problem 9.

a. Prove: $(P \vee (Q \wedge R)) \wedge (R' \vee S) \wedge (S \rightarrow T') \longrightarrow (T \rightarrow P)$

b. Write the following as a predicate wff, and then prove it using predicate logic. The domain is all people.

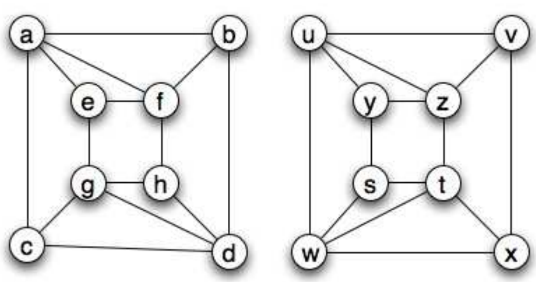
   Every CEO speaks only to Vice-Presidents, and some CEO speaks to someone. Therefore there is a Vice-President. C(x), S(x,y), V(x)

Problem 10. Draw a tree whose traversals are

- in-order: e,b,j,i,h,a,c,g,d,f, and
- pre-order: a,b,e,h,i,j,c,d,g,f.

Then write the post-order traversal.

Problem 11. Decide whether these two graphs are isomorphic or not, justifying your answer:



Problem 12. Prove by induction the formula $\sum_{i=1}^{n} i^2 = \frac{1}{6}n(n+1)(2n+1)$

Problem 13. Recall the binary search algorithm: starting with an element to search for (the "search element") and a sorted list (you may assume its length is $2^n - 1$ for some positive integer n), compare the middle element to the search element; if not equal (and we'll assume that the search element is **not** in the list), search either the right or left half (i.e. recurse), depending on the relative positions of the middle element and the search element.

a. Why did I suggest that you start with a list of length $2^n - 1$?

b. Write the number of comparisons $C(2^n - 1)$ as a recurrence relation (assume the worst case). What is the base case?

c. Find the number of comparisons you must do in the worst case.

Problem 14. Illustrate the following types of graphs:

a. All distinctly different (non-isomorphic) trees with five vertices (put root at the top – you might organize the trees by height).

b. All complete graphs up to six vertices. Which are planar?