MAT385 Test 1 (Spring 2013): 1.1-1.4; 2.1, 2.2

Name:

Directions: Problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1: (10 pts)

a. (5 pts) Use propositional logic to prove the following (use these statement letters to write it as a propositional wff: G – glaciers melt, M – Miami disappears, T – Temps rise, C – we dump carbon):

The Greenland glaciers melt only if Miami disappears. If the Greenland glaciers don't melt then the temperature didn't rise. If we continue to dump carbon into the atmosphere, the temperature will rise. We continue to dump carbon into the atmosphere. Therefore, Miami will disappear.

b. (5 pts) Use any method to demonstrate that $A \wedge B' \rightarrow (A \rightarrow B)'$.

Problem 2: (10 pts)

a. Consider the argument $(P \to Q) \land (P' \to Q) \to Q$.

i. (2 pts) Explain this wff in words.

ii. (5 pts) Prove it, using propositional logic.

b. (3 pts) Prove it with a truth table.

P	P'	Q				

Problem 3: (10 pts) Exactly one of these wff is valid:

a. $(\forall x)[P(x) \vee Q(x)] \rightarrow [(\forall x)P(x) \vee (\forall x)Q(x)]$

b. $(\forall x)[P(x) \vee Q(x)] \rightarrow [(\forall x)P(x) \vee (\exists x)Q(x)]$

Prove the valid wff (you may prove it in any equivalent form), and provide an interpretation which invalidates the other.

Valid:

Invalid:

Problem 4: (10 pts)

a. (5 pts) Prove that this wff is valid:

$$(\exists y)(\forall x)\{P(y) \wedge [Q(x) \rightarrow R(x,y)]\} \rightarrow (\exists y)\,\{P(y) \wedge (\forall x)[Q(x) \rightarrow R(x,y)]\}$$

b. (5 pts) Using predicate logic, first state the following as a wff in predicate logic, and then prove it valid in the domain of all farm animals (use predicate wff names $EG(x), G(x), EA(x)$):

Some farm animals eat only grass. Goats eat anything. If an animal eats anything, then it eats more than grass. Therefore, there is a farm animal that is not a goat.

Problem 5: (10 pts) Let n be an odd natural number. That is, it can be written as $2m + 1$ for some other integer $m \geq 0$. Consider the following theorem:

If n is an odd natural number, then its square can be written as $8k + 1$ for some integer k.

a. (5 pts) Prove the theorem by any means (you may assume a theorem due to Gauss).

b. (3 pts) What is the converse of this theorem? Is the converse of this theorem true?

c. (2 pts) What is the contrapositive of this theorem? Is the contrapositive true?

Problem 6: (10 pts) Use proof by induction to demonstrate $P(n)$, for $n \geq 1$, where

$$P(n): \quad 1 + 3 + 5 + ... + (2n - 1) = n^2$$