MAT385 Test 2 (Spring 2013): 2.4-5.2

Name:

Directions: Problems are equally weighted, as are subparts at the same depth. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1: (10 pts)

a. A set S of strings is defined recursively by

   i. a and b belong to S.

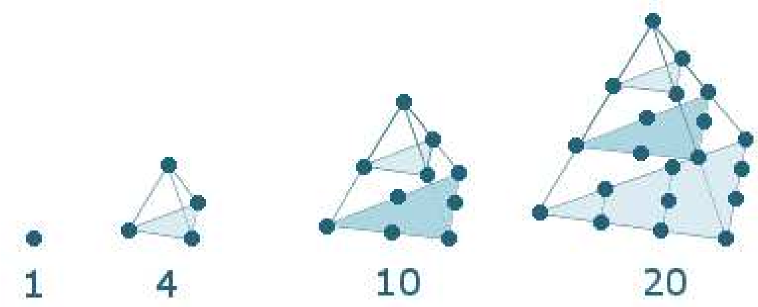
   ii. If X belongs to S, then so do the concatenations Xb and aX.

   Which of the following belong to S? (Show how each is formed, or why it can't be formed.)

   - ab
   - aba
   - $aaab$
   - In words, completely characterize the strings in S.

b. Give a recursive definition of the complete graphs K_n. (What are the essential elements of a recursive definition?)

Problem 2: (10 pts) Find and solve a recurrence relation for the n^{th} tetrahedral number, $T(n)$. The first few – $T(1)$ through $T(4)$ – are shown here:



Hints:

- The tetrahedral numbers are built up from Gauss's triangular numbers $G(n)$ $(1, 3, 6, 10, 15, \ldots)$. You may need to first find a closed-form solution for those.
- $\displaystyle\sum_{i=1}^{n} i^2 = \frac{1}{6}n(n+1)(2n+1)$.
- FYI: $\displaystyle S(n) = c^{n-1}S(1) + \sum_{i=2}^{n} c^{n-i}g(i)$

Problem 3: (10 pts)

a. Define the power set $P(S)$ of a set S. Illustrate with the set $S = \{a, b, c\}$ as an example.

b. Suppose set S has n elements. Explain why (better yet, prove that) the power set has 2^n elements.

c. Make the case that the set S of strings of Problem 1 is countably infinite (define "countably infinite").

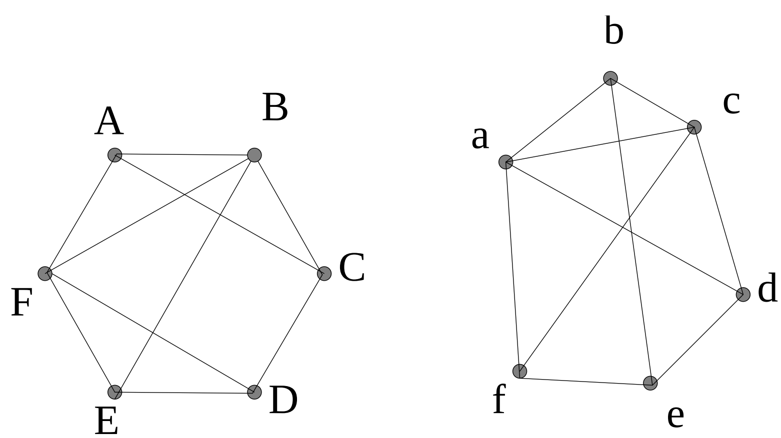
Problem 4: (10 pts)

a. Illustrate the following types of graphs:

   i. All distinctly different (non-isomorphic) trees with five vertices (put root at the top – you might organize the trees by height).

   ii. All simple graphs with three vertices.

b. Decide (and justify) whether or not the following graphs are isomorphic. Hint: planar graphs and $K_{3,3}$ (the houses and utilities graph).



Problem 5: (10 pts)

a. Draw a tree with the traversals prescribed below (and you fill in the post-order):

- pre-order: g,h,i,d,a,e,c,b,f
- in-order: d,i,h,a,g,c,e,b,f
- post-order:

b. Draw the expression tree for ([(x*3)+2]-[y/(7+x)])*4. Then represent the tree using the left child/right child array. Number the vertices of the tree from 1 to 13 by depth, and from left to right (0 is null).

	left child	right child
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		