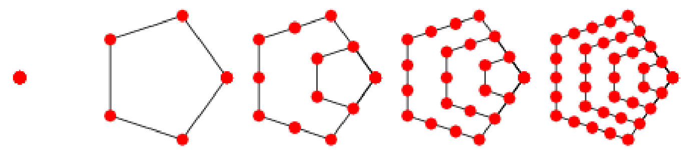
MAT385 Test 2 (Fall 2016): 2.5-5.2

Name:

Directions: Problems are equally weighted, as are subparts at the same depth. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1: (10 pts)

a. (6 pts) Find a recursive formula for the n^{th} pentagonal number, P_n. The first few – P_1 through P_5 – are shown here (think new "corner" and "side" points; how is P_5 related toP_4?):



b. (4 pts) Then find a closed-form solution for P_n, simplifying your answer as much as possible.

Problem 2: (10 pts)

a. (4 pts) Demonstrate binary search for the element "2" on the list $(0, 1, 3, 5, 7, 8, 9)$.

b. (6 pts) Write a recursive formula for comparisons using binary search on a sorted list of length $n = 2^m$, for m a natural number. Then find a closed-form solution, simplifying your answer as much as possible.

Problem 3: (10 pts)

a. (3 pts) Describe how we can determine if two sets are the same size, even if the sets are infinite.

b. (3 pts) Write the power set of the set of binary arithmetic symbols $S = \{*, -, +\}$. (How can you check that you have found them all?)

c. (4 pts) Decide whether the following two proposed "binary operations" are indeed binary operations on the given set:

i. $x \circ y = xy$ (concatenation on the set of finite strings of symbols from the set S above).

ii. $x \circ y =$ common divisor of x and y; $S = \mathbb{N}$.

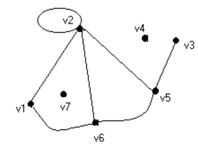
Problem 4: (10 pts)

a. (5 pts) Shown below at right is a sample graph with seven **labelled** vertices and seven arcs.

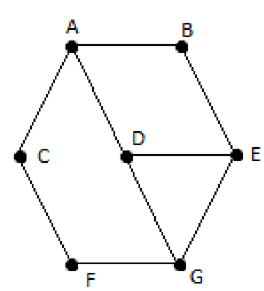
   i. Draw all non-isomorphic graphs with four (note: **unlabelled**!) nodes and two edges (I find seven).

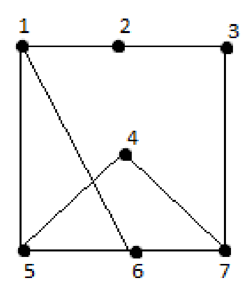
   ii. Which are simple? Mark simple graphs with an "S".

   iii. Hint: two pairs have the same sets of degrees, but are non-isomorphic – point them out.

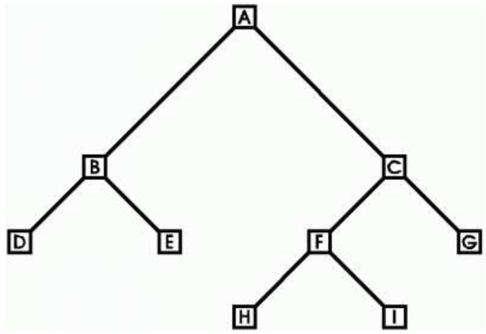


b. (5 pts) **Prove** that the following graphs are isomorphic by finding a bijection between the sets of nodes (Why do you only need **one** bijection?):





Problem 5: a. (10 pts) Consider the following tree:



b. (4 pts) Write the list of nodes resulting from the following traversals:

- pre-order:
- in-order:
- post-order:

What advantage(s), if any, does post-order confer over the other two?

c. (4 pts) Give both an array representation of the tree and an adjacency matrix representation (you may use a dot $-\cdot-$ for "missing" entries). Why would you prefer one over the other?

	left child	right child
A		
B		
C		
D		
E		
F		
G		
H		
I		

	A	B	C	D	E	F	G	H	I
A									
B									
C									
D									
E									
F									
G									
H									
I									

(2 pts) There are "missing" entries in the array representation above. What can you say about the relationship between the number missing, m, and the number present, p, for a tree with n nodes?