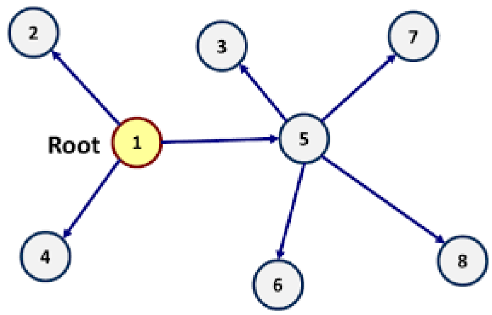
MAT385 Test 2 (Spring, 2017): 2.6, 3.1, 5.1, 5.2, 5.3

Name:

Directions: **You must skip one problem**: write "skip" prominently on it. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1: (10 pts) Consider the following tree:



a. (3 pts) Write the list of nodes resulting from the following traversals (**order children in increasing numerical order**):

- pre-order:
- in-order:
- post-order:

b. (4 pts) At right, draw the expression tree for $(3 * x - 7) * (1/(x - 3))$. Write the expression in both

- prefix notation

- postfix notation

c. (3 pts) Draw a tree that has postorder traversal $d, a, h, e, i, b, f, g, c$ and preorder traversal $c, i, d, e, a, h, g, b, f$.

Problem 2: (10 pts) Consider all possible "Facebooks" that could exist between four distinct individuals.

- (4 pts) How many different friendships are possible? Draw a graph representing the Facebook where all four are friends with the others. What's the name of this graph, and why is its name appropriate?

- (3 pts) How can we use the power set of some set to create all possible Facebooks for these four individuals? How many different Facebooks are possible?

- (3 pts) Draw all possible Facebooks that are trees or forests (collections of trees), ignoring distinctions of individual. In other words, draw all distinctly different trees and forests with exactly four nodes.

Problem 3: (10 pts) We've done some problems with detecting counterfeit coins (one lighter than the others) with a balance scale. There is a difference in how one approaches the problem depending on whether the number of coins is even or odd.

a. Describe your first step if the number of coins n is even.

b. Describe your first step if the number of coins n is odd.

c. What is the worst case number of weighings if $n = 2^m$, where m is a natural number? Write and solve the recurrence relation (it's easy).

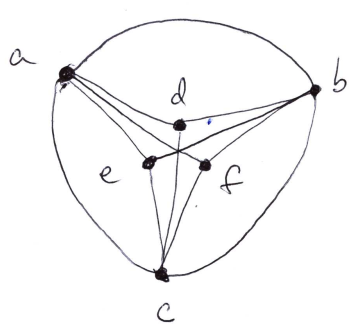
d. What is the depth of the decision tree if the number of coins is $n = 2^m - 1$?

Problem 4: (10 pts)

a. (4 pts) All non-planar graphs contain a subgraph isomorphic to one of two graphs: which graphs are they? Draw them and name them.

b. (6 pts) **Prove** (quite simply) that

   i. the graph below is not planar; and



   ii. if we remove any one vertex from the graph, it IS planar.

Problem 5: (10 pts) Prove that for any finite set S the power set $P(S)$ is larger in size that S itself. (I might suggest a particular mapping and contradiction; induction would also work.)

Problem 6: (10 pts) I'll give you this much of a reminder of the Euclidean algorithm: $a = qb + r$; then recurse. When $r = 0$, b is the greatest common divisor of the original two numbers.

a. (4 pts) Illustrate the Euclidean algorithm for the computation of the greatest common divisor of the two **successive** Fibonaccis 21 and 34. What is special about the choice of consecutive Fibonaccis with respect to this algorithm?

b. (3 pts) Now consider the Euclidean algorithm for the computation of the greatest common divisor of the two **non-**consecutive Fibonaccis 34 and 13 (with just one "skip" – 21). What do you observe about the very first remainder?

c. (3 pts) Prove that this pattern always works with "almost consecutive Fibonaccis", those with only one Fibonacci between them: $F(n)$ and $F(n-2)$.