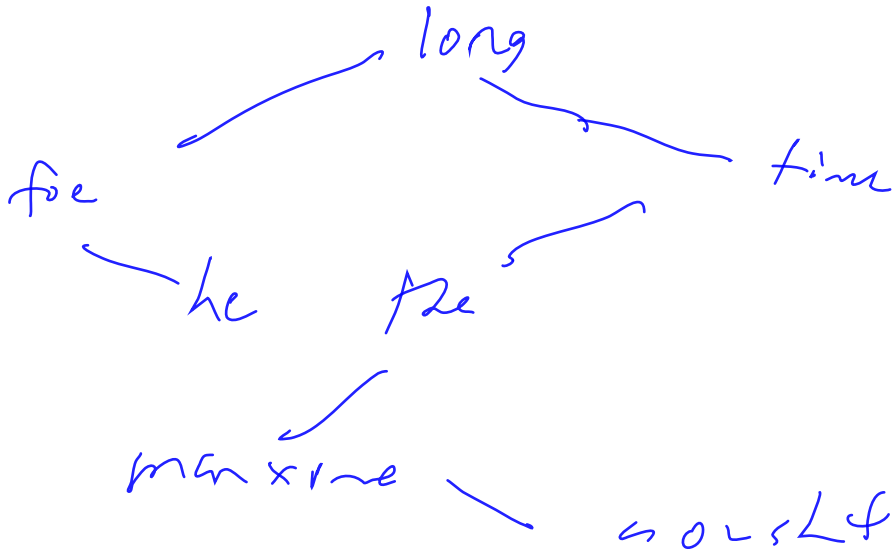
MAT385 Test 2 (Spring, 2020): 4.1, 6.1, 6.2, 6.3

Name: Key

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1: (17 pts)

a. (4 pts) Create a binary search tree by entering this line of Lewis Carroll's poem "Jabberwocky" in order: "long time the manxome foe he sought".



b. (9 pts) Write the first lines of the new "poems" resulting from the following traversals:

- (1 pt) in-order: foe he long manxome sought the time
- (1 pt) pre-order: long foe he the manxome sought
- (1 pt) post-order: he foe sought manxome the time long

i. (1 pt) What's curious and special about the in-order traversal "poem"?

It's in alphabetical order.

ii. (1 pt) What's the worst case number of comparisons for a binary tree search for a word not on this list? (And provide a word that would trigger it.)

A word that would trigger it is any word strictly between manxome & the, not equal to sought.

Five comparisons.

iii. (2 pts) What's the **best** worst case number of comparisons, if we had entered the data in a different order? (How many nodes fit in a binary tree of depth d?)

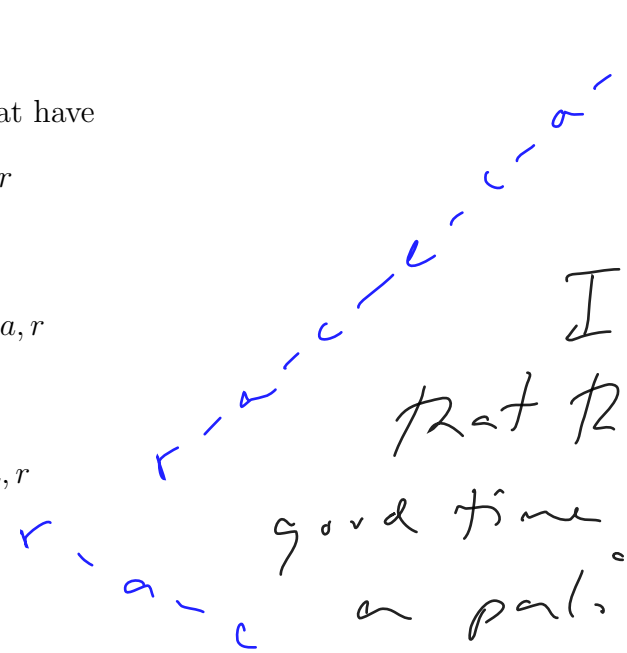
7 words → binary

depth 2 should do → 3 comparisons

iv. (2 pts) Create a more "balanced" binary search tree by adding the words in a better order.

c. (4 pts) Draw two different trees that have

- inorder traversal r, a, c, e, c, a, r
- postorder traversal r, a, c, e, c, a, r
- preorder traversal r, a, c, e, c, a, r



I thought that this was a good time to draw in a palindrome....

A good way to approach a problem like this is to think of doing it for a simpler palindrome - e.g. "pop": ...

Problem 2: (12 pts) Consider the set of three colors, $S = \{R, G, B\}$ (red, green, blue), which we use to create any "RGB" color.

a. (4 pts) How many different colors can we create by combining (or not) each of the three (imagine adding some subset of these colors to white paint)? (Assume no gradations of color – either in or out – all or none.)

Possibilities: $S = \{R, G, B\}$,
$\{R, G\}, \{R, B\}, \{G, B\}$ 8
$\{R\}, \{G\}, \{B\}$ Colors
$\emptyset$ (White!)

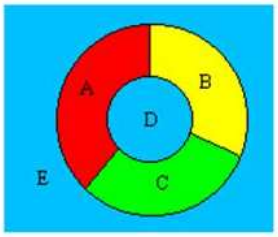
b. (2 pts) For a set S of n elements, what is the size of the powerset $P(S)$?

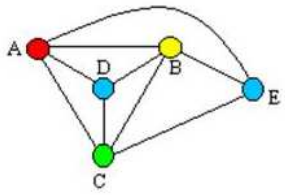
2^n

c. (2 pts) What can we say about the size (cardinality) of the powerset of an infinite set S?

It's infinite; a bigger flat of S.

d. (4 pts) We certainly have enough colors to color any map, since we only need four:





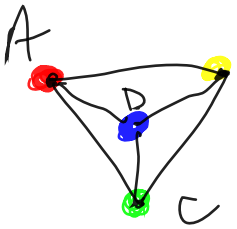
Planar graph - a graph drawn in a plane without any of its edges crossing or intersecting

Each vertex (A,B,C,D,E) represents a region in a graph

Each edge represents regions that share a boundary

i. (3 pts) The image above shows five countries requiring four colors, as well as the corresponding graph – nodes are countries, arcs are boundaries. It's not the **simplest** map that requires four colors, however. Pare this map down to one still requiring four colors, and draw the corresponding graph.

Draw out D or E



Remove any arc & you can get by with 3 colors

ii. (1 pts) **Name** and describe the simplest graph that **requires** four colors.

K_4 - the complete graph on 4 vertices.

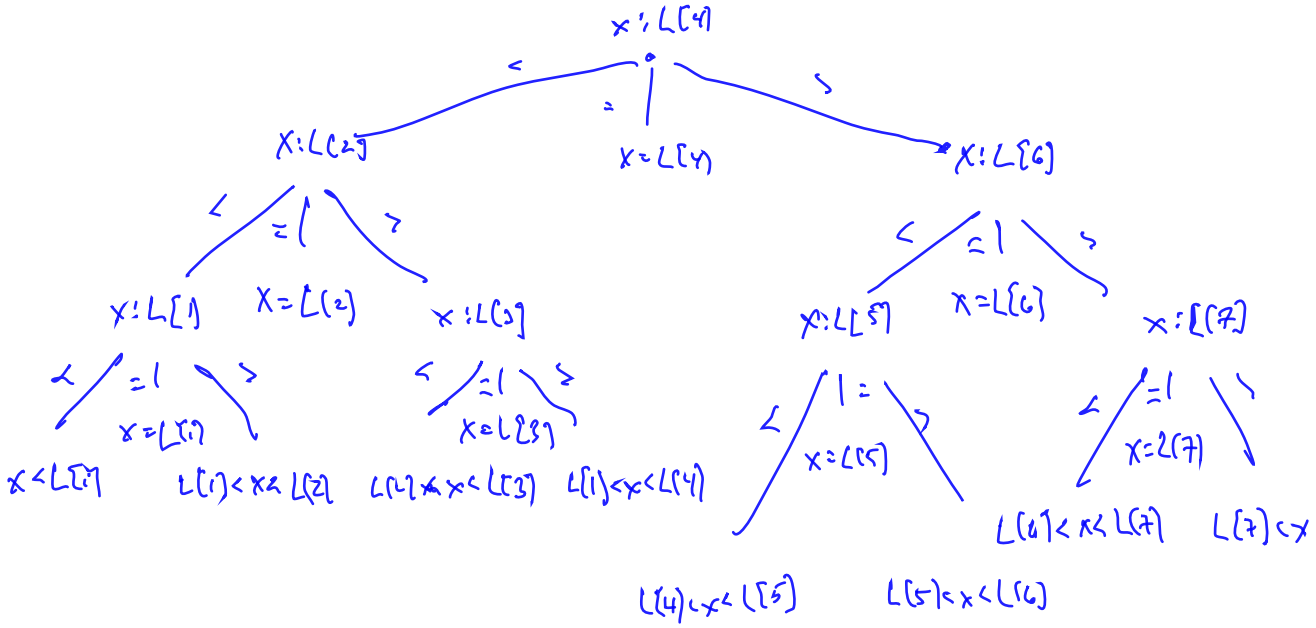
Problem 3: (11 pts) We've done some problems with binary search on various sorted lists. There is a difference in how one approaches the problem depending on whether the list length m is even or odd.

a. (1 pt) Describe your first step if the list length m is odd.

Go to the middle element & compare — if smaller, go to the "middle" of the sublist to the left; otherwise go right, to the "middle"...

b. (9 pts) Consider a sorted list of length $m = 2^n - 1$, where n is a positive integer, and a binary search of this list. We are going to consider the decision tree where internal nodes represent comparison of x to an element of the list, and leaves represent the three possibilities: $<, >, =$.

i. (5 pts) Draw the ternary decision tree for $n = 3$ (i.e. binary search on $m = 7$ elements).



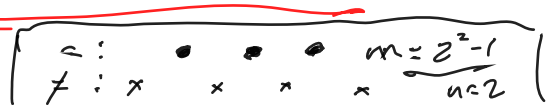
ii. (3 pts) How many leaves (different possible reported results) are **necessary** in the general case $2^n - 1$? (How many different ways could the search end? How many endings – leaves – are there for $n = 3$ in your tree above?)

$2^n - 1$ $=$: 7 ways

$2^n - 1 + 1$ $\neq$: 8 ways

15 total ways:

$2^{n+1} - 1$ leaves

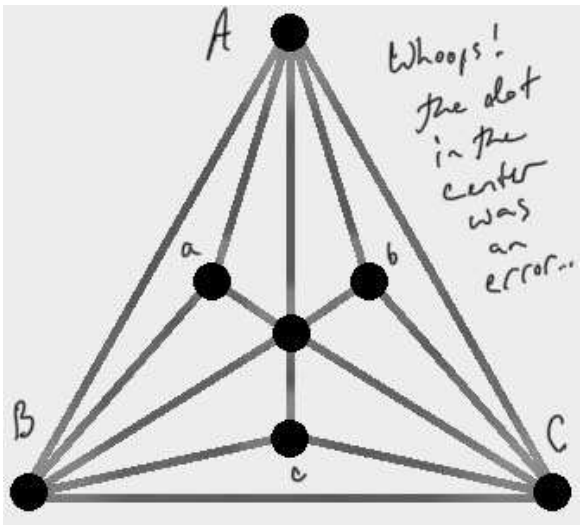


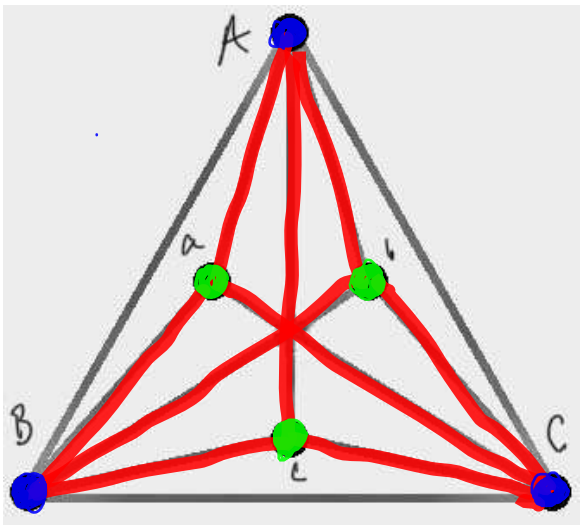
iii. (2 pts) What is the depth of the ternary decision tree for any $2^n - 1$? What is the worst case number of comparisons? (Why do I ask both these questions on the same line?:)

Depth in this case is 3; 3 comparisons are required in the worst case. So for $m = 2^n - 1$ the depth of the tree is n.

Problem 4: (10 pts)

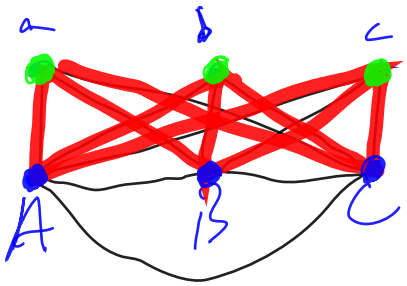
a. (5 pts) The beautiful graph at left is clearly planar. At right is drawn another graph, where I have





eliminated the dot in the center – because it was a mistake. There wasn't supposed to be a node there – just false intersections. Demonstrate that the graph at right is **non**-planar. (Hint: it contains a famous non-planar graph as a sub-graph. You might redraw the graph to show this.)

If a graph contains a subgraph of K_5 or $K_{3,3}$, it's not planar. (Kuratowski's Theorem)



$K_{3,3}$

b. (5 pts) **Prove** (quite simply) that if we remove any one node from the graph at right, it IS planar. (Use **symmetry** – you don't need to make six arguments; or use a theorem about graphs with only five nodes.)

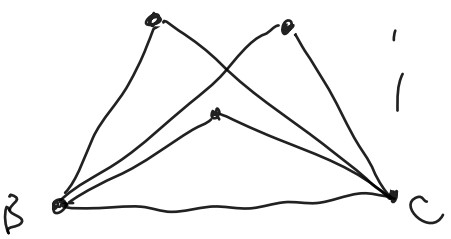
OR

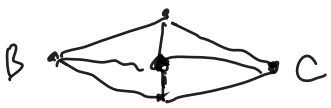
Invoke Theorem

To fail to be non-planar it must contain a subgraph of K_5 or $K_{3,3}$ (six nodes). It has only 5 nodes, & no K_5

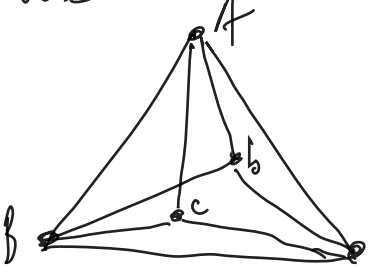
Symmetry + cases:

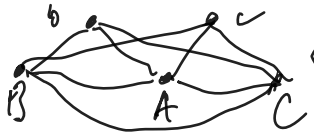
Remove A





remove a





← contains copy of $K_{2,3}$, planar

but that's

Academic Honesty Statement

At the moment, the university finds itself in unprecedented times. Faculty have worked to create exams that mimic the traditional face-to-face testing environment, and we are asking students to respond with honesty and integrity during this public health crisis. I have no doubt that you will respond in this way, but my role as your teacher also requires that I remind you of the Student Honor Code in the Code of Student Rights Responsibilities:

https://inside.nku.edu/scra/information/students/rights-responsibilities.html

The work you submit on this exam is bound by this policy. Below are some examples of behavior that would violate this code:

- Using unpermitted resources while completing the exam. This includes searching for answers or approaches on the Internet. It also includes asking (calling, texting, facetiming, etc.) your classmates, friends, relatives, acquaintances, Siri, Alexa, or Google for assistance on the exam. Basically, if it's a behavior that your instructor would not allow while taking an exam during a traditional class meeting, you should also not do this during the online test.
- Submitting an exam completed by another person (either in whole or in part). Do not complete the exam and then share your responses with others. That would allow them to submit work that was not their own, and both individuals will be deemed in violation of the Honor Code.
- Discussing the exam with others in the course. When you submit your exam, you should not discuss the test with anyone else in the class until the due date has expired – for everyone. This includes texting and/or calling someone and asking if they've taken the test – that feels like an innocent question, but it can quickly become a conversation that violates the bullets above. During the exam window (basically throughout the day), you should only communicate with the instructor of the course, but no one else enrolled in the class.

Before submitting your exam, you will be asked if you have satisfied the Student Honor Code. Please respond with honesty and integrity.

When grading your work, if it is deemed you have violated the Student Honor Code, then all future exams must be completed with the use of Lockdown Browser with webcam monitoring. It will be your responsibility to ensure you have the proper technology to allow for this to occur. Additionally, the score earned on the exam in question will not be used when calculating your course average. I will communicate via email how this impacts the calculation of your grade in the course.

You are now be asked to sign the following statement, provided it is true:

The work submitted for this exam was completed in a way that did not violate NKU's Student Honor Code.

Signature:

Date: