Section Summary: 1.7
Limits and Continuity

a. Definitions

Let f be a function of x defined on some open interval that contains the number a, except possibly at a itself. Then we say that the **limit of** $f(x)$ **as** x **approaches** a **is** L, and we write

$$\lim_{x \to a} f(x) = L$$

if, for every number $\epsilon > 0$ there is a corresponding number $\delta > 0$ such that if

$$0 < |x - a| < \delta$$

then

$$|f(x) - L| < \epsilon$$

In plain English, what this says is that when x approaches a from either direction, then the function values get closer and closer to L.

A function f of x is called **continuous at** a if

$$\lim_{x \to a} f(x) = f(a)$$

We say f is **continuous on** D if f is continuous at every point a in D.

b. Theorems

All polynomials are continuous on $\Re$. All rational functions are continuous on their domains. More generally, sums and products of continuous functions are continuous on their domains.

If we approach a from right and left and get two different limits, then the limit

$$\lim_{x \to a} f(x)$$

fails to exist.

c. Properties/Tricks/Hints/Etc.

Typical problems in the univariate case are

- vertical asymptotes
- holes
- step functions

d. Summary

It's nice when we can invoke continuity of functions (e.g. sum of continuous function is continuous on the mutual domain, etc.). Rational functions are continuous on their domains.