MAT112 Section Summary: 7.1

Antiderivative

Summary

We've been taking derivatives of functions: from a function, we create a new function, called the derivative. Now we want to undo that, just like we use inverse functions to undo one another (think of logarithms and exponential functions). If you're given a function f, can you interpret it as a derivative? That is, can you find a function F that has that function f as its derivative? So here's the deal:

Before: given $f(x)$, find $f^{'}(x)$.

Now: given $F^{'}(x)$, find $F(x)$. Only we're going to be given $f(x)$; think of it as a derivative, $f(x) = F'(x)$; and try to find F.

Now it turns out that this is harder than it seems: the process of finding derivatives is very mechanical, and it turns out that the inverse process is not so mechanical. Let's see how far we can get....

Antiderivative: If $F'(x) = f(x)$, then $F(x)$ is an antiderivative of $f(x)$.

Note that we say "an" antiderivative: it turns out that it's not unique. Note also that we're using small letters for functions we're thinking of as derivatives, and capitals for those we're thinking of as antiderivatives.

If $F(x)$ and $G(x)$ are both antiderivatives of a function $f(x)$ on an interval, then

$$F(x) = G(x) + C$$

for some constant C.

So if you can find an antiderivative F, then you can write them all as $F(x)+C$, where C is an arbitrary constant. This represents a *family* of functions. We have a special (and quite unusual) notation for this family:

$$\int f(x)dx$$

is called the **indefinite integral** of f; the symbol $\int$ is called the **integral sign**, and $f(x)$ is called the **integrand** of the integral. So

$$\int f(x)dx = F(x) + C$$

for any real number C. The x inside the integral is called a dummy variable: so

$$\int f(t)dt = F(t) + C$$

works just as well!

Some examples:

1. #5, p. 392

2. #7, p. 392

3. #21, p. 392

As in the case of derivatives, we can write up a bunch of general rules for integrals:

1.
$$\int kf(x)dx = k\int f(x)dx$$

2.
$$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$$

3. Power rule:
$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

4.
$$\int e^{kx}dx = \frac{e^{kx}}{k} + C$$

5.

$$\int x^{-1}dx = \int \frac{1}{x}dx = \ln|x| + C$$

Some examples:

1. #46, p. 392
2. #51, p. 392
3. #58, p. 393