MAT112 Section Summary: 7.3

Area and the Definite Integral

Summary

As if by total surprise, we begin considering the problem of calculating an area. While this may seem like a disappointing end for those of you going on to study business, you should be aware that this area problem is just the easiest and most intuitive example of integration. Lots of business calculations can be thought of as "areas under a curve", so never fear! This really is practical....

Example: #24, p. 413

As you were warned, the indefinite integral will turn into a definite integral, when we put limits on the integral sign:

The Definite Integral: If f is defined on the interval $[a, b]$, the definite integral of f from a to b is given by

$$I = \int_a^b f(x)dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x,$$

provided the limit exists, where $\Delta x = \frac{b-a}{n}$ and x_i is any value of x in the i^{th} interval. a is called the **lower limit** of integration, and b is called the **upper limit** of integration.

Example: #3, p. 411

You can think of the sum on the right-hand side as representing little rectangles, whose

1. locations along the x-axis are given by x_i,
2. heights are given by $f(x_i)$, and
3. widths are given by Δx.

Now you see that the dx in the integral is not merely indicating the variable of integration, but has a direct analogy with distance – it really is important! It's playing the role of the Δx in the sum, so it represents a width of a rectangle. $f(x)$ is also representing a height: it's just that we're computing an infinite number of wafer thin (actually thinner!) rectangles, located at x of height $f(x)$ and width dx, as x varies from a to b.

The definite integral can be approximated by

$$I \approx \sum_{i=1}^{n} f(x_i)\Delta x,$$

and will be when your calculator can't figure out how to compute the solution exactly, using formulas from calculus. We'll talk next time about the Fundamental Theorem of Calculus, which tells us how to compute these exactly (when we can!).

Often the choice of the locations x_i is made according to some scheme. Several popular schemes are

1. left-endpoints,
2. right-endpoints, and
3. mid-endpoints.

Each has its advantages. Mid-points tends to give a more accurate approximation.

Example: #9, p. 412

Example: #15, p. 412