The Monty Hall Dilemma Revisited:

Understanding the Interaction of

Problem Definition and Decision Making

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ABSTRACT

We examine a logical decision problem, the “Monty Hall Dilemma,” in which a large portion of sophisticated subjects appear to insist on an apparently wrong solution. Although a substantial literature examines the structure of this problem, we argue that the extant analyses have not recognized the constellation of cues that guide respondents' answers. We show that insight into subjects' decisions may be obtained by considering problems with similar surface structure to the Monty Hall Dilemma but which are common in environments that they routinely face. In particular, we consider the problem modeled as a game in which actors have possibly opposing interests, and as an environment with information provided by an objective source. We present experimental evidence showing that these comparisons help to explain subject responses.

Analyses of decision making behavior frequently focus on decision problems in which subjects violate normative standards. One of the most baffling puzzles of the past decade is the "Monty Hall Dilemma," named after the host of the television game show "Let's Make a Deal." A large share of even sophisticated subjects tenaciously insist on an apparently wrong solution to this problem. We may summarize the problem, as follows:

> A contestant is informed that an automobile is behind one of three doors and gag prizes (goats) are behind the other two. After the contestant makes a tentative choice, say door 1, the host opens one of the other doors, say door 3, showing that it does not contain the prize. The contestant is then given the option of deciding to stick with her original choice or to switch to door 2. She wins the automobile if it is behind the door she chooses. Is it in the contestant's interest to switch doors?

In this form, the puzzle dates to Selvin (1975a, 1975b), and appeared in Nalebuff (1987). After its nationwide syndication in vos Savant (1990a), more than 40 papers focusing on this problem appeared in academic journals and the popular press (see Barbeau, 1993, for an extensive but still incomplete list). If rather than asking whether the contestant should switch doors, the subject is asked what the chance is of winning when the contestant sticks with door 1 or switches to door 2, the problem is conceptually similar to the "Three Prisoners Problem," which dates to the 1950s (Gardner, 1959a, 1959b).

The solution given by those who pose the problem is that the contestant should switch doors. In contrast, when confronted with this puzzle, an overwhelming majority of subjects indicate that the contestant should stick with her original choice. The assertion that switching doors is correct appears to be counterintuitive even to those with extensive statistical training.

The most common explanation for why the contestant should switch is as follows: If the contestant sticks to her original choice, her chance of winning must be 1/3. Once an unchosen, incorrect alternative has been revealed, the automobile must be behind one of the two remaining doors, so the chance that the automobile is behind the remaining door that was not originally chosen must be 2/3.

The treatment below begins by providing a general formal solution that incorporates current commentary. This problem has proven difficult for laymen and professionals alike because the underlying process by which the host makes his decisions is critically important in determining the correct answer, but that process is only understandable in terms of events that do not occur in the narrative. None of the extant analyses has recognized adequately or completely the constellation of cues in this problem which guide respondents' answers in the face of this difficulty.

We argue that several related models provide insight into the answers subjects give. We first consider the problem as an economic game, in which the host and the contestant both seek to make optimal choices. We also point out that the problem shares important elements with a circumstance in which information is provided by an objective source, a problem which admits to a simple quantitative answer.

We then present experimental data indicating that comparison of these cases may help to explain the answers that subjects give. If the Monty Hall Dilemma is difficult, in part, because it resembles a game in which individuals face an opponent, then the host's incentives would be relevant to observed answers. Similarly, if the difficulty is due to problems in distinguishing the Monty Hall Dilemma from ostensibly similar puzzles whose underlying structure differs, we would expect subjects to provide answers that do not differ across puzzle types. Our experiments examine how subjects' responses vary by host intent and by information provided to subjects about the underlying problem structure.

The Monty Hall Dilemma is representative of a class of problems in which individuals use widely reliable cues which, under the given circumstance, are misleading. Although such behaviors appear, on their face, to violate rational standards, they are explicable in terms of a rational, well-defined decision making strategy. This is not to say that individuals' actions are, in fact, narrowly rational, since we recognize that responses are shaped by both the cognitive limitations of the individual and the particulars of personal experience.

THE HOST'S PROTOCOL AND THE GENERAL SOLUTION

The statement of the problem provided above specifies almost nothing about the host's protocol. Most important, it does not specify whether the host considers the position of the prize in deciding whether to open a particular door or whether he is required to open a door under all circumstances. Instead, the statement of the problem specifies one instance of his behavior that is consistent with a large number of possible protocols. The answer that the contestant has a 2/3 chance of winning if she switches follows if we assume that the host's protocol requires that he reveal an incorrect, unchosen door regardless of whether the contestant's initial choice was correct, and that, if two doors are available, the host opens them with equal probability. In addition, it assumes that the prize is behind each door with equal probability. None of these assumptions is specified in the problem as it is commonly stated.

In order to characterize the problem's solution, we extend the notation of Morgan, Chaganty, Dahiya, and Doviak (1991) to allow for these possibilities. We limit our consideration to the case where the contestant initially chooses door 1, since the host's protocol in cases where the contestant initially chooses doors 2 or 3 can be specified analogously. Let the probabilities that the automobile is behind doors 1, 2, and 3 be denoted p_1, p_2, and p_3. Let p_{ij} (i=1, 2, 3; j=0, 1, 2, 3) indicate the host's protocol, identifying the probability that the host opens door j when the prize is behind door i, with j=0 indicating he does not open a door. As an example, the probability that the host will open door 2 when the automobile is behind door 1 is p_{12}, and the chance that he does not open a door is p_{10}.

The probability of winning (W) for the contestant who chooses to stick with her original choice, given that a goat has been revealed behind door 3 (G3), is given as

$$\Pr(\mathrm{W}|\mathrm{Stick,G3}) = \frac{p_1 p_{13}}{p_1 p_{13} + p_2 p_{23}}, \quad \textbf{(1)}$$

while that for the contestant who switches is

$$\Pr(\mathrm{W}|\mathrm{Switch,G3}) = \frac{p_2 p_{23}}{p_1 p_{13} + p_2 p_{23}}. \quad \textbf{(2)}$$

The solution normally provided assumes that p_1=p_2=p_3=1/3, p_{23}=p_{32}=1, and p_{12}=p_{13}=1/2, or, in words, that there is an equal chance that the automobile is behind each door, that if the automobile is behind doors 2 or 3, the host chooses the door without the prize, and if the prize is behind door 1, the host chooses doors 2 and 3 with equal probability. In this case, the probability of winning by sticking is 1/3, and the probability of winning by switching is 2/3. However, this solution is not necessarily implied by the narrative as it is commonly stated. The problem statement specifies only that the host has opened door 3 to reveal a goat, implying no more than $p_1 p_{13} + p_2 p_{23} > 0$.

Morgan et al. (1991) were the first to provide a formal analysis showing that the answer normally given by those stating the puzzle relied on unstated assumptions. They pointed out that if the host favored either door 2 or door 3 when the prize was behind neither (so that $p_{12} \neq 1/2$), or if there was a chance he could open up the door containing the prize (i.e., $p_{22}>0$ or $p_{33}>0$), the probabilities of winning depended on the host's particular protocol.

The assumption that the host follows a protocol in which he must offer the contestant the chance to switch is also critical. If the host were to choose to open a door and provide the opportunity for the contestant to switch when, and only when, the contestant's original choice was correct, any contestant who chose to switch when given the opportunity would lose every time. Such a strategy for the host amounts to setting $p_{10}=0$, and $p_{20}=p_{30}=1$, while maintaining $p_{ii}=0$, $i=1, 2, 3$.[1]

LITERATURE EXPLAINING HOW RESPONDENTS SOLVE THE MONTY HALL DILEMMA

In addition to the attempts to explain the formal structure of the Monty Hall Dilemma, a literature also attempts to provide a behavioral explanation for observed choices by individuals facing the Monty Hall Dilemma. This literature is rooted in the "Three Prisoners Problem," which we repeat here:

> Three prisoners, Tom, Dick and Harry, face potential death sentences. They know that two of the three will die, and one will be pardoned. The warden, who knows who will be pardoned, has been instructed that he cannot let any prisoner know if he is to be pardoned. In the absence of any knowledge, each prisoner must assume that his chance of being pardoned is 1/3. Dick approaches the warden and convinces him that by naming either Tom or Harry as one who will be executed, he will not be violating his instructions. The warden names Harry. Dick now rejoices: He calculates his chance of being pardoned has increased to 1/2, since either he or Tom must be pardoned. Is his reasoning correct? What is the chance that Tom is the one to be pardoned?

Early published versions of this puzzle appeared in Gardner (1959a, 1959b, 1961) and Mosteller (1965). Corresponding to the Monty Hall Dilemma, those who pose the question indicate that the correct answer is that Dick's chance of a pardon remains at 1/3, while Tom's increases to 2/3. The formal structure of the Three Prisoners Problem differs in several respects from that of the Monty Hall Dilemma. First, rather than making a decision about a course of action, the subject is estimating a probability. Second, the problem statement makes clear that the warden cannot reveal which prisoner has been pardoned, nor can he reveal Dick's fate to Dick. Finally, it specifies that he knows who will be pardoned, which is not specified in many versions of the Monty Hall Dilemma.[2]

Unlike the Monty Hall Dilemma, the Three Prisoners Problem explicitly focuses on the paradox inherent in the conclusion that Dick's chance of execution appears to be reduced regardless of what the warden tells him. Many respondents conclude that the chance that Dick is to be pardoned must remain at 1/3, corresponding to the accepted answer. However, many respondents conclude that the chance of execution must be equal for Dick and Tom, and so specify a chance of 1/2. As in the Monty Hall Dilemma, the problem provides only information about how the warden behaves in one instance, rather than indicating his detailed protocol. If the warden's decision of whether to name a prisoner or of which one to name depends on whether Dick is pardoned, then the probabilities are, in fact, indeterminate.

A small literature, much of it in Japanese, recognized the ambiguity in the warden's protocol. Several explanations for respondents' choices were based on the possibility that respondents believed the warden would favor Harry or Tom when he responded to Dick's request to name a prisoner who had not been pardoned.[3] Ichikawa and Takeichi (1990) reported on experiments which attempted to test these explanations by specifying, or requiring subjects to specify, the warden's protocol. They concluded that respondents were not making choices that could be explained by such beliefs.

Experimental results suggested that subjects' responses could be explained as the application of "subjective theorems" or heuristic rules (Shimojo & Ichikawa, 1989; Ichikawa, 1989). Subjects who recognized that there had been no change in Dick's chance of execution, upon learning about Harry, appeared to be applying a version of the "irrelevant therefore invariant" theorem. This idea reflects the view that since Dick knew that either Harry or Tom would be identified by the warden, their names would be irrelevant and could not provide him with additional information. Even in cases where the protocol was explicitly specified so that the warden favored Harry or Tom, with the result that this rule does not provide the correct answer, these subjects responded in accord with this rule. In contrast, those subjects who stated that both Dick and Tom had a 1/2 probability of a pardon appeared to be following the "uniformity assumption" (Falk, 1992) reflecting the view that each option had an equal chance of occurrence.[4]

Falk's (1992) analysis considered both the Three Prisoners Problem and the Monty Hall Dilemma, implying that the two problems were formally identical. She showed that the subjective theorems used in these problems have an extensive history of both appropriate and inappropriate application. There is substantial evidence consistent with the view that the strategies employed in the Monty Hall Dilemma correspond to those in the Three Prisoners Problem, although usage rates are different. In the Monty Hall Dilemma, the overwhelming majority of subjects choose not to switch doors, and their explanations are very often consistent with the uniformity assumption. It appears that those with statistical or mathematical training are often convinced that this is the only possible correct answer (Morgan et al. 1991; vos Savant, 1990b; Shaughnessy & Dick, 1991). The brilliant mathematician Paul Erdös reportedly insisted on this answer, having difficulty understanding why switching was the better choice even after observing a computer simulation (Schechter, 1998, pp. 108-109). In contrast, for those who claim that switching is the better choice (including most published explications), the most common explanation for this choice accords with the “irrelevant therefore invariant” theorem.

Granberg and Brown (1995), and Granberg and Dorr (1998) performed experiments testing whether subjects were able to make better choices when faced with repeated instances of the Monty Hall Dilemma. Although they found evidence of learning, many subjects chose to stick with an original choice even after many trials. Their explanations for observed patterns focused on psychological theories of counterfactual thinking and regret, which would make respondents reticent about changing an initial choice even when it was appropriate to do so.

Friedman (1998) also examined the extent to which subjects learned due to repetitions of the Monty Hall Dilemma. He argued that the apparently irrational choice was best viewed as a "pseudo-anomaly," reflecting transient behavior in an unfamiliar environment. He conjectured that rational behavior would ultimately dominate as learning occurred.

Our analysis of respondent solutions differs from these in several important ways. The evidence is clear that respondents do solve complex problems, like the Monty Hall Dilemma, with reference to a variety of strategies. The mistakes they make are very often associated with use of the kinds of subjective beliefs that have been identified by Shimojo and Ichikawa (1989), Ichikawa (1989), and Falk (1992). Individuals also follow rules which, under some circumstances, are consistent with regret or counterfactual thinking, as suggested by Granberg and Brown (1995) and Granberg and Dorr (1998). It seems likely as well that violations of rational behavior will tend to subside given sufficient time and training (Friedman, 1998), although it is not clear what it takes to obtain thoroughly rational choices.

In contrast to Friedman, our focus is not on the structure of learning but on explaining observed behavior. The critical question, we believe, is how individuals choose, in any one circumstance, which rule, or combination of rules, to apply. Our contention is that the individuals use a set of complex cues that are, in turn, drawn from their own experience and knowledge. As a result, in order to explain observed behavioral patterns which appear to deviate from prescribed standards of rationality, it is necessary to examine how a particular task relates to other more common tasks faced by individuals.

DIFFICULTY RECOGNIZING THE ROLE OF THE HOST'S PROTOCOL

Given all the interest in the Three Prisoners Problem beginning in the 1960s, it is notable that we have not found a single instance where a published version of the puzzle fully specified the warden's protocol. The narrative makes clear that the warden has the option of choosing not to respond to Dick's request for information. If his choice of whether to respond depends in any degree on whether Dick has been pardoned, the very fact that he is provided with a name can allow Dick to better estimate his probability of survival. Analogous to the Monty Hall Dilemma, it is easy to construct protocols for the warden in which the chance that Dick is pardoned takes any value between zero and one. Not only is the warden's behavior not specified, but published answers to the problem often do not specify the necessary assumptions for the answer they provide.[5]

Statements of the Monty Hall Dilemma published until the early 1990s are similarly defective, and most published answers do not fully explicate the necessary assumptions. The discussions in Tierney (1991), Morgan et al. (1991), Engel and Venetoulias (1991), and Gillman (1992) were the first to recognize the importance of fully specifying the host's protocol. In response, recent statements of the problem (Piattelli-Palmarini, 1994; Granberg & Brown, 1995; Granberg & Dorr, 1998; Friedman, 1998) often do make clear that the host is constrained to open an unchosen incorrect door, although in some recent cases the narrative continues to suffer the defect of earlier versions.[6]

None of the analyses that attempt to explain the pattern of subject responses places much importance on this aspect of the host's protocol. Rather, these treatments implicitly follow the lead of popular authors who appear to view the detailed specification of the host's actions as a technical matter of concern to those who focus on the problem's formal structure. As an example, vos Savant responded to those who criticized the ambiguity in her narrative: “Pure probability is the paradigm, and we published no significant reason to view the host as anything more than an agent of chance *who always opens a losing door and offers the contestant the opportunity to switch*” (1991, page 347, italics added). Of course, the problem description she presented, which corresponded closely to the one in the introduction to our paper, did *not* in fact provide sufficient detail for a reader to unambiguously infer this information.

The contention here is that the host's protocol provides a key to understanding how subjects make choices in the Monty Hall Dilemma. The published literature alerts us to the fact that even trained statisticians have difficulty seeing the critical role of the host's potential actions. It should be no surprise that most respondents are unable to recognize this as well.[7]

We suggest that the reason that both naive subjects and trained statisticians do not pay attention to these matters is because, in the repertoire of problems that they routinely face, they have learned to attend to a different set of cues. If we assume that individuals will attend to cues that help them in making apparently similar decisions, the questions of interest are: What other kinds of decisions might individuals face that would appear similar? What cues are likely to be relevant? Bar-Hillel and Falk (1982) point out that in order for a subject to correctly solve such problems implies the construction of an underlying model of the relevant statistical process. Individuals will use pieces of the problems they have observed in the past to construct the models on which to base their judgments.

We turn next to two models representing environments commonly faced by individuals but which share important surface elements with the Monty Hall Dilemma. In each case, we argue that subject responses to the Monty Hall Dilemma can be understood in terms of the optimal responses in these cases.

Strategic Information Provided by a Knowledgeable Adversary

Individuals frequently find themselves in circumstances where they are playing a game with a knowledgeable adversary. Whereas it may be difficult to sort out the details of the host's protocol in the Monty Hall Dilemma, there is little doubt that he is knowledgeable and that he may have a substantial stake in the outcome. These are strong cues, and it is reasonable to assume that the respondents' choices may be shaped by the adversarial structure implicit in the narrative: If the contestant wins the automobile, the host may bear part of the cost.

What is the subject's rational response in the case where the host follows a strategy that minimizes the chance of winning? To answer this question, we need to consider the interaction of the strategies chosen by both the host and the respondent. The Nash equilibrium identifies those combinations of strategies for the host and contestant in which each is making the best choice given the choice that the other makes. Consider the case where the contestant initially chooses door 1, and where the chance that the automobile is behind each door is 1/3. For the host, the set of optimal strategies are those in which $p_{12} \geq p_{32}$, and $p_{13} \geq p_{23}$. These conditions imply that the host offers the choice of switching relatively more frequently when the prize is behind door 1. The contestant's optimal strategy is to stick with her original choice if the host reveals that a door not chosen contains a goat (G2 or G3). The intuition for why the contestant should always stick to her original choice in a Nash equilibrium is straightforward. If she follows a policy in which she ever chooses to switch, the host will offer the choice to switch only when the contestant's original choice was correct. The contestant always loses in this case, so the decision to switch cannot be part of a Nash equilibrium. A formal characterization of the Nash equilibria appears in Appendix A.

The Nash equilibrium is relevant if contestants face observationally similar circumstances in which such a game may provide the appropriate model. One possible model is the game show, "Let's Make a Deal," in which Monty Hall actually provided contestants with opportunities that closely paralleled our puzzle. Since the show was broadcast for decades (including reruns), popular knowledge might well incorporate information about the game as it was actually played. If the host always revealed a losing alternative, we might anticipate that observers would learn that switching provided a greater chance of winning. In fact, in the actual show, the opportunity to switch was provided on some occasions but not on others. The tension of the game revolved around the motives of the host, so that the ambiguity of his protocol played an important if implicit role in the show.[8] In short, insofar as subjects choose their responses to our puzzle with reference to the actual game show, they are unlikely to view the host as a disinterested "agent of chance," the characterization that vos Savant (1991) suggested was appropriate.

More generally, Nalebuff (1988) described the strategy of avoiding bets against knowledgeable others with reference to the advice that the gambler in *Guys and Dolls* received from his father:

> Son, one of these days in your travels a guy is going to come to you and show you a nice brand-new deck of cards on which the seal is not yet broken, and this guy is going to offer to bet you that he can make the jack of spades jump out of the deck and squirt cider in your ear. But son, do not bet this man, for as surely as you stand there you are going to end up with cider in your ear. (p. 152)

Information from an Objective Source

An alternative environment is one in which information is provided without regard for which option was chosen. Consider the Problem of the English Philosopher, which can be stated as:

> You come to a 3-way fork in the road. You know only one of the roads goes to your intended destination, Bath, but you have no sense of which road it is. Nor do you have any other information that would allow you to distinguish between the roads. An English philosopher appears on the scene and makes an offer: "I will write the destination of each road on a piece of paper. After you make an initial choice among the forks, I will let you pick one at random to help inform your final choice." After deciding on road 1 as your initial choice, you pick a piece of paper and find that road 3 does not go to Bath. Should you now switch your choice to road 2?

In this case, since the chance a road identified is equal for all three roads, and is made without regard for the true road, we have simply p_{ij} = 1/3, for i, j=1, 2, 3, and p_{i0}=0. Using the expressions in Equations 1 and 2, it is clear that the odds associated with doors 1 and 2 are not influenced by the information about door 3. If the subject was initially indifferent between roads 1 and 2, she will be indifferent between the two choices at this point.

The English philosopher's actions differ from those of the host in the Monty Hall Dilemma in several respects. One critical distinction is that the information the English philosopher provides is not influenced by the choice made by the subject, so that the information received might identify an initial choice as incorrect. The subject's original choice is therefore irrelevant. If the subject has no information that allows her to distinguish among the three alternatives, symmetry requires that the odds for any pair must remain unchanged when a third option is removed from the choice set. The same conclusion applies if the English philosopher allows the subject to obtain information only about a path other than the one originally chosen, a structure formally identical to that considered by Nickerson (1996).

Where information is provided by an objective source, the Problem of the English Philosopher is the appropriate model for rational decision making. Insofar as subjects interpret the host's actions in the Monty Hall Dilemma in this way, they will conclude that the odds associated with doors 1 and 2 remain unchanged. Switching is never the better choice, and it is inferior if there is even the slightest initial preference for the first choice. In the Three Prisoners Problem, this structure would imply that Dick and Tom have equal chances of a pardon based on the information the warden provides.

Overview of Experiment

In order to provide relevant evidence for examining these explanations, we administered questionnaires asking subjects to indicate the best response in various versions of the Monty Hall Dilemma. We varied the details of the host’s protocol across subjects, in some cases fully specifying the host’s protocol and in other cases allowing substantial latitude, in order to examine whether subjects were responsive to such details. In addition to asking subjects whether it was best to switch doors, we asked them to estimate probabilities of winning and to provide the reasoning for their answers. Since the first model above implies that subjects act as if they believe the host has interests that oppose theirs, in some versions of the problem we explicitly informed the subject of the host's interests. Finally, to further investigate when subjects were responsive to host intent, we examined a variant of the Monty Hall Dilemma in which the host offers to make an additional side payment.

METHODS

We administered questionnaires to 328 undergraduate college students who volunteered to participate for extra course credit. Each student provided answers for four puzzles relating to the Monty Hall Dilemma. The overall experimental design was complex, and, in total, some 15 versions of the puzzle were included in questionnaires in various orders. However, embedded in the design were several fully balanced experimental structures, which facilitate tests of hypotheses. Further detail on the experimental design appears in Appendix B.

In each case, the problems were hypothetical, and subjects were given no feedback about the appropriateness of their answers. Although it is often important to provide incentives in studying laboratory behavior, there is no evidence that incentives play any role in producing paradoxical behavior in the Monty Hall Dilemma.[9] As a rule, experimental environments that do not provide incentives or the opportunity for learning display greater random behavior, so we would expect our design to militate against the hypothesis that patterns would reflect rational considerations.

Subjects received one of three basic problem versions, each with a conceptually distinct logical structure but corresponding, in surface presentation, to the Monty Hall Dilemma. In all three, instructions indicated a prize behind one of three doors with the subject given the option of switching her choice from door 1 to door 2 after the host reveals that door 3 does not contain the prize. The subjects were, in each case, asked to indicate the probability of winning after sticking with door 1 and the probability of winning after switching to door 2. They also were asked whether it was best to stick or switch. All three problems began with the passage:

> A thoroughly honest game-show host has placed a car behind one of three doors. There is a goat behind each of the other doors. You have no prior knowledge that allows you to distinguish among the doors.

The problem types differed in the statements that followed. The first version of this problem fully explicated the host's protocol:

> "First you point toward a door," he says. "Then I'll open one of the other doors to reveal a goat. After I've shown you the goat, you make your final choice whether to stick with your initial choice of doors, or to switch to the remaining door. You win whatever is behind the door."
>
> You begin by pointing to door number 1. The host shows you that door number 3 has a goat.

We label this the *Monty Hall Standard*. Although this is not the version that is most common in the literature, it makes unambiguous the assumptions that are necessary for justifying why switching is the best choice. Given this protocol, the chance of winning for someone who sticks with the original door is 1/3, while the chance of winning for someone who switches is 2/3.[10]

The second version does not specify the host's protocol, but merely indicates one instance of it:

> You begin by pointing to door number 1. The host shows you that door number 3 has a goat. He says to you, "Now that I've shown you the goat, you can make your final choice whether to stick with door 1 or switch to door 2."

We label this *Monty Hall Ambiguous*. This corresponds closely to the version of the Monty Hall Dilemma as it is most widely stated. In order to calculate whether it is best to switch, some assumption about the host's protocol must be made. If a contestant follows a policy of sticking with the original choice, the chance of winning is 1/3, whereas a policy of switching will yield a chance of winning that can vary between 0 and 1, depending on the host's protocol.

The third version of the problem fully specifies the host's protocol as one in which he chooses randomly among all three doors but, in the particular case at hand, has chosen door 3. This corresponds to the Problem of the English Philosopher specified above.

> "First you point toward a door," he says. "Then I'll open one of the three doors at random. After that, you make your final choice whether to stick with your initial choice of doors, or to switch to one of the other doors. You win whatever is behind that door."
>
> You begin by pointing to door number 1. The host shows you that door number 3 has a goat.

We label this *Monty Hall Random*. Here, the chance of winning is 1/2 for sticking with door 1, and 1/2 for switching to door 2. The fully informed contestant should be indifferent between sticking with the original door and switching.

The three versions indicated above were varied across subjects. Although each subject responded to three problems, all problems faced by a given subject were of the same basic type. In contrast, host intent was varied within subjects. Where the problem was as stated above, with no modification, we refer to the host as displaying *Neutral Intent*. Where we inserted a statement indicating that the host wishes to reduce the chance of the respondent winning, we label this *Negative Intent*, and where we indicated that the host wishes to increase the chance of the respondent winning we label this *Positive Intent*. For those respondents who were given a

Neutral Intent problem, it was always the first problem in the questionnaire. In contrast, Negative Intent and Positive Intent problems could appear in positions 1-4.

Finally, in some problems we added a statement that the host offers an extra $100 to the respondent if she chooses to switch (*Money to Switch*). This variation appeared only in positions 3 or 4.

The instructions specified that subjects were to answer questions in order and that under no circumstance should they change an answer to a prior question even if they believed they had made a mistake. After responding to the four problems, subjects were asked to provide explanations for their answers.

RESULTS

We first examined how subjects responded to the three basic problem versions in the Neutral Intent condition. We then examined the impact of purported host intent and then the impact of monetary incentives. Finally, we considered subjects' estimates of the chance of winning and how they explain their responses.

Responses to the Three Problem Types

Our initial question was whether a substantial number of subjects responded differently based on the host's protocol. If respondents were able to correctly solve each problem, they would switch when facing the Monty Hall Standard problem, and they would be indifferent between switching and sticking when facing the Monty Hall Random problem. In contrast, their response to the Monty Hall Ambiguous problem would depend on assumptions about the host's protocol, which was not fully specified, so any answer could be justified.

The difficulty that trained statisticians have in recognizing the critical assumptions in the problem suggests that there will be very few subjects who recognize a difference between the Standard and Ambiguous versions. Given that even those with statistical training frequently claim that the chance of winning should be 1/2 whether the subject sticks with the original choice or switches, it seems unlikely that there will be a substantial difference between responses to the Standard and the Random versions.

In order to get a clear measure of differences in responses to the three basic problems, we examined the problems in the Neutral Intent condition and in which the host offered no money. Since neutral intent problems appeared only in position 1, responses are not contaminated by other conditions. Table 1 shows that the proportion switching varied from 11 percent for the Monty Hall Ambiguous problem, to 18 percent for the Monty Hall Random problem and 19 percent for the Monty Hall Standard problem. These proportions are not statistically different from one another (Chi-square=1.26, df=2, p>.05) and they are within the range reported by others who have administered the Monty Hall problem to a variety of populations (Granberg and Brown, 1995; Friedman, 1998).[11] The lower proportion switching in the Monty Hall Ambiguous version suggests that some subjects may recognize that the host has more latitude in that case, but the evidence is weak. There is no indication that subjects are able to distinguish the Standard and Random versions of the problem.

One may ask whether previous experience with the Monty Hall puzzle would alter answers subjects might give. After responding to the problems, subjects were asked if they had heard or read previously about any of the four problems or any similar ones. Of the 326 respondents who answered this question, 9 percent answered yes. Although the number is too small to undertake separate detailed analyses by problem type or condition, tabulations of their answers reveal that there is much variation in answers within this group, and that very few provide fully "correct" answers. For example, of the five subjects who reported having knowledge of the puzzle and who answered the Monty Hall Standard problem (Neutral Intent) as the first problem, only two specified that switching was the best option, the correct answer given in published versions of the problem. However, when we aggregated all Monty Hall problems, combining all conditions, we found that those individuals who said they had knowledge of the problem were more likely to choose to switch when given the chance. We have repeated the analyses below omitting these individuals and have found no substantial differences in results.

The Impact of Intent

Do subjects respond to host intent? In the Standard and Random cases, host intent can have no impact on the chance of winning. In contrast, host intent may be important in the Monty Hall Ambiguous case insofar as it provides information about the host's protocol. When the host acts to reduce the contestant's chance of winning, the Nash equilibrium is an appropriate model to predict behavior. We noted above that in this case the Nash equilibrium implies that the contestant never should switch from the original choice. In contrast, when the host is interested in helping the contestant win, the Nash game suggests that switching may be help the contestant win (see Appendix A). Hence, given this reasoning, we expect that the Negative Intent condition would induce less switching than Positive Intent, but only for the Monty Hall Ambiguous case.

We limit consideration to those problems in which the host made no offer of money. Such problems occurred in positions 1-3. Initial investigation showed that position was important, and so we fitted a model which allowed for the effects of basic statement, intent, and position. Table 2 provides coefficient estimates based on a logit specification in which the dependent variable is the log odds of the probability of choosing to switch from an initial choice. Note that coefficients are constrained to sum to zero across all categories for each variable.

When statement type, intent and position are included in the model, only the effect of position is statistically significant. We see that subjects are substantially less likely to choose to switch on the first problem than they are on later problems, but positions 2 and 3 do not differ. Theory provides no obvious single reason for what is clearly a strong effect,[12] but it is consistent with Friedman's (1998) finding that subjects were more likely to switch in later plays of the game even after measures designed to capture learning had been controlled. We tested for remaining interaction effects but did not find any that were statistically significant.

The strong impact of position and the complex experimental design suggest the possibility that our attempt to control for position may not have been successful. We therefore undertook several analyses based on a balanced design that removes any position effects. First, we examined the effects of statement type and intent for problems in position 1. The analysis was limited to Monty Hall Standard and Monty Hall Ambiguous, since, by design, only neutral intent problems were available in position 1 for the Monty Hall Random case. We found that effects of statement type and intent were small and not statistically significant. This is consistent with the earlier finding of few differences in response to problems in position 1 (Table 1).

The study design allows for an analysis of problems in positions 2 and 3 that is fully balanced by position. This analysis uses data only from questionnaires in which the first problem was a Neutral Intent problem, while problems in positions 2 and 3 were Negative Intent and Positive Intent, with their ordering reversed in half the cases. An analysis based on problems in positions 2 and 3 in such questionnaires fully controls for position and order effects. Table 3 presents a model that examines the effects of basic problem type and host intent, first in the case requiring that intent have the same effect for all basic problem types (Model 1) and then allowing the effect of intent to be different for the Monty Hall Ambiguous case (Model 2). Problem position had no impact and its effects are not included. Since Neutral Intent problems always appeared in position 1, this analysis limits consideration to Negative Intent and Positive Intent.

Although Model 1 in Table 3 indicates that neither problem type nor intent is statistically significant, Model 2 shows that when the effect of intent is permitted to differ for the Monty Hall Ambiguous problem type, effects become statistically significant. In the Monty Hall Ambiguous condition, subjects are much less likely to switch when the host is attempting to prevent them from winning the prize than when he wishes to help them. There is also a main effect, showing that subjects are significantly less likely to switch in the Monty Hall Ambiguous Condition than in the other two conditions. Table 4 (No Money condition) presents the simple percentage switching for the problems used in this model, allowing a more direct examination of these effects. Subjects facing the Monty Hall Ambiguous problem are nearly three times as likely to switch in the Positive Intent condition as in the Negative Intent condition (30 percent versus 11 percent), whereas there is no substantial difference for the other basic problem types. These results are consistent with the view that 70 percent of subjects choose to stick with an original choice in all problems and 10 percent chose to switch in all problems. The remaining 20 percent recognize that it is valuable to switch in those problems with a fully defined protocol or with an ambiguous protocol if the host is attempting to help them win.

Taken together, these results indicate that even though subjects respond similarly to the three basic types of problems when they first face them, when they are provided with information on host intent, some do distinguish in just the way that rational choice would imply. Notice that for the problems we have analyzed here, intent is varied within subjects, so subjects are cued to the potential importance of intent. However, all variation in basic problem type is between subject, so that differential response by problem type has not been cued by within subject variation.

The Impact of Side Payments and Intent

In the previous section, we showed an impact of host intent only for the Monty Hall Ambiguous problem type. This pattern suggests that at least some subjects implicitly recognize differences in problem type, since a correct analysis would imply that intent is relevant only for the ambiguous version. In order to further examine how host intent influences choices, for many subjects we included a problem in which the host offered a side payment if the contestant switched. This problem occurred in the third or fourth position, after subjects had already responded to problems indicating both positive and negative intent. The Money to Switch condition was applied to a problem that was identical to the one immediately preceding it but specified that the host offered $100, in addition to whatever prize was won, for switching. The host's offer occurred after the contestant had made her initial selection, so that, for any of the problem types, the offer could be contingent on the initial choice.

The rational response to such monetary incentives includes two effects. The first is the simple value of the $100, which, other things equal, should cause those close to indifference to decide to switch. The second effect results from any information content that the offer of money may have. None of the three problems specified that the offer of money was part of the host's pre-specified protocol. It therefore could be rational for the subject to make inferences about the position of the prize based on whether the monetary incentive was provided. In such cases, the host's intent would likely be relevant.

The simplest model would suggest that the host who wishes to reduce the contestant's chance of winning (Negative Intent) would offer money to switch in the case where the original choice was correct. The offer of money would therefore cause a contestant who was aware of the host's incentives to not switch. In contrast, where the host wished to increase the chance of the contestant winning (Positive Intent), it is plausible that the offer of money to switch would be taken as a hint that switching would win the prize. So we predict a strong impact of host intent in the case where he offers the contestant money to switch.[13]

Table 5 presents estimates from two models designed to test the main hypotheses developed above. We limited consideration to problems with Negative Intent and Positive Intent, since monetary incentives were only provided in these cases. While the structure of the model presented in the table is dictated by our hypotheses, we also tested for higher order interaction effects and found none that were statistically significant.

Model 1 of Table 5 presents estimates based on the sample of all problems using positive and negative intent. Since the unbalanced design implies that position is correlated with problem type and intent, position must be controlled. Positions 3 and 4 are combined because the effect of position 4 is not identified independent of the other effects we wish to examine. The specification allows the impact of intent to differ for the Monty Hall Ambiguous problem, and also allows its impact to differ by whether money is offered. Estimates show that money exhibits a strong main effect, with those offered money being much more likely to switch, regardless of question type or intent. Results also indicate that the impact of intent is much greater where money has been offered, confirming the hypothesis that subjects correctly recognize that the host's offer of money will provide information that depends on his own interests.

Model 2 of Table 5 presents estimates based on questions that together make up an experimental design that is balanced by order and position.[14] Only problems in positions 2-4 are included in this analysis. The estimates based on this sample are very similar to those based on all questions taken together. An important advantage of this design is that effects presented in the model are captured by simple tabulations of the proportion switching by problem type, intent and monetary incentive. Table 4 provides tabulations on which this model is based. The number switching increases appreciably when the host offers monetary incentives, but the impact is much stronger when the host displays positive intent. The number choosing to switch approaches 80 percent in the Monty Hall Standard and Monty Hall Random cases with positive intent and monetary incentives.

These results confirm that subjects respond to monetary incentives. It is hardly surprising that they are more likely to choose to switch when they are offered money for doing so. Equally important, the impact of intent shows that they are acutely aware of the incentives faced by the host, and how these are likely to influence host behavior. The subtlety of the patterns of subject response suggests that even where the full understanding of the statistical issues underlying the problem is beyond almost all respondents, the resulting patterns tend toward the predictions of the rational model.

Estimates of Chance of Winning

In addition to asking subjects whether they would choose to switch doors, our questionnaire asked respondents the probability of winning if they stuck with their original choice and if they switched. The modal answer for both questions was 1/2, with approximate 80 percent of subjects giving this answer in the neutral intent condition (position 1). In the Monty Hall Standard problem, the chance of winning will be 1/3 for those who stick and 2/3 for those who switch. Only 8 percent of respondents in that condition indicated a probability of 1/3 as the chance of winning after sticking, while fewer than 2 percent gave the 2/3 answer. For conditions in which money was offered, the proportion giving the 1/2-1/2 answer declined to as little as 50 percent of subjects.

The simplest analysis would suggest that subjects' probability estimates would respond to the conditions much as did choices but would not respond to the offer of money in the same way. We performed an Analysis of Variance taking as the dependent variable the difference between the subject's reported chance of winning after switching and the chance of winning after sticking. Rather than present coefficient estimates, Table 6 presents the mean values of the dependent variable for the subset of questions forming a balanced design, which fully capture estimated effects. As an example, the entry .071 for the Monty Hall Standard-Negative Intent-No Money condition indicates that subjects judged the chance of winning for switching to be .071 higher than for sticking. The patterns across question type and host intent for difference in reported chance of winning correspond closely to those involving the choice of whether to switch (Table 4). In the No Money condition, intent matters only for the Monty Hall Ambiguous problem type, whereas intent is relevant in all cases where money is offered to switch.

The main effect of money, however, is appreciably different for Table 6 than in the earlier tables that focus on the decision to switch. When offered money, the average relative estimate of the chance of winning after switching does not increase, but rather decreases in the case of negative intent (compare columns identifying No Money-Negative Intent with Money to Switch-Negative Intent) and increases in the case of positive intent. This confirms that subjects are responsive to how host incentives to offer side payments influence their chance of winning, even though they also clearly make choices reflecting the direct benefits of the money.

Since a large portion of subjects indicated equal chances of winning for sticking and switching, the above analysis can only provide a partial explanation for why people stick with their original choice. Table 7 shows the percentage who say they would switch among those individuals who report equal probabilities. As we might anticipate, for those individuals who believe that the chance of winning is the same for the two choices, an additional payment of $100 will cause many to choose to switch an initial choice. On the other hand, the impact of intent on subject choice indicates that the choice process is not as simple as it might appear. Table 7 shows that where the host's protocol is ambiguous, even those subjects who believe that the odds of winning are even are substantially more likely to switch if the host's intent is positive. Apparently, for many subjects, the probability estimate does not capture all that is relevant.

How Subjects Explain Their Answers

In open-ended responses to the question of how they estimated the probabilities, the overwhelming proportion of subjects gave the simple answer that since the prize was necessarily behind one of two doors, the odds must be equal, so the chances were 50 percent at both. For problems in which no money was offered, the proportion giving this answer varied from 75 to 90 percent when the problem appeared in position 1, and decreased to around 60 percent for problems in later positions. The proportion giving this answer did not depend on host intent or basic problem type in any clear way. Not surprisingly, for those problems in which the host offered a monetary incentive, the proportion giving this answer declined further, to about 50 percent.

Despite the obvious importance of this simple answer, the open-ended responses do support the notion that many subjects are conscious of host intent. Since few subjects mentioned intent for problems in position 1, Table 8 reports how subjects viewed host intent in open-ended responses for problems in positions 2-3, without monetary incentives, which are balanced for all order effects. Depending on the problem, up to a quarter volunteered that host intent had influenced their estimate of the chances, while up to a quarter specifically said that it did not. As might be anticipated from the analyses reported above, subjects who commented on host intent were much more likely to indicate that intent mattered for the Monty Hall Ambiguous problem than for the Monty Hall Standard or Monty Hall Random problems.

Although the request for an explanation focused on estimates of probabilities, approximately half of the open-ended responses made an explicit reference to the decision of whether the subject should choose to switch. The most common explanation for sticking with the original choice in the problems with no monetary incentives, given by up to half of all respondents who gave any explanation for their choice, was a statement to the effect of, "My first choice is usually right."

For problems with monetary incentives, the structure of the responses changed quite dramatically. In both the Positive Intent and Negative Intent conditions, up to half of subjects gave a response indicating that they had chosen to switch for the money. Many of these responses suggested that they had no way to determine which door contained the prize so that winning the $100 was the best they could expect. Another set of answers put substantial importance on the host's intent. Up to a third of those who provided an explanation in the Negative Intent condition with monetary incentives indicated that they chose to stick because the offer of money to switch indicated that switching would lose the prize. In the Positive Intent condition, a similar number of answers indicated that the offer of money suggested that they would win if they switched.

There were a substantial number of idiosyncratic answers. These included inferences about how the prize would be placed based on host actions and the specific numbers of the doors, and attempts to infer the position of the prize from various ideas about host behavior.

We also asked subjects whether the problems were like decisions they had faced in real life, excepting statistics problems or puzzles. About half gave an answer. Although most indicated a variety of situations which share little more than large rewards for a correct decision, approximately a quarter of those providing an answer compared the problems to deciding whether to switch answers on a multiple choice exam. Of course, here information about an unchosen alternative is most likely to be provided by an objective source, as in the Problem of the English Philosopher. In this problem, the odds involving any remaining alternatives are unchanged when information about an unchosen alternative is provided. The proportion giving this answer did not differ among the three basic problem types.

DISCUSSION

In the Monty Hall Dilemma, why do subjects so often choose to stick with their original choice, and why do they obstinately insist that the chance of winning is 1/2 whether they stick or switch? The surface answer is quite simple: Modal responses to our open-ended questions imply that a large share of subjects state that they believe their first answer is usually the best one, an indication of belief perseverance or counterfactual thinking. A large proportion add that where there are two choices, one of which is a winner, the odds must be even, apparently reflecting the uniformity assumption. It would appear that we have observed the application of simple rules that have led subjects astray.

Even if we accept these subjects' explanations at face value, they do not provide a real understanding of the answers people give to the Monty Hall Dilemma. Our analysis makes clear that the application of these rules is contingent on context in subtle ways. Responses are often sensitive to host intent, which has no obvious role in the subjective theorems or heuristic rules used to describe subject behavior. Of course, we can explain such patterns of behavior by expanding the set of rules, but we believe that this approach will do little more than proliferate rules to be applied in an *ad hoc* fashion across environments.

An alternative view is that subjects' choices in the Monty Hall Dilemma are best explained in terms of the cues it shares with decision environments they routinely face. We have argued that the surface structure of the Monty Hall Dilemma has elements in common with environments in which individuals face a knowledgeable adversary, and those in which they receive information from an objective source (e.g., the Problem of the English Philosopher). Minor changes in the host's protocol can produce a structure that corresponds with either of these, and in both it is appropriate to stick to an original choice. Since the published record shows that the importance of the host's detailed protocol is lost on even sophisticated decision makers, we argue that the decision to stick with an original choice may be interpreted as rational given subjects' analytical limitations.

Our experiments confirm that when subjects first encounter the Monty Hall Dilemma, their decisions to stick with an original choice is not responsive to the host's detailed protocol. Our reading of the evidence is that cues associated with both alternative problems contribute to explaining this choice. The cues implying an objective source of information cause subjects to place equal probabilities on the two outcomes. It is suggestive that many subjects said the choice was much like the decision to change answers on a multiple choice exam, although presumably the original choice would not usually be random in that case.

At the same time, cues of a knowledgeable adversary appear relevant, since subjects stick with their original choices even if they state that there is no difference in the chance of winning. The decision to stick with an initial choice is not merely a mechanical reaction, since we see that subjects are conscious of strategic considerations. In the Monty Hall Ambiguous condition, about a fifth of subjects recognized, at least implicitly, that switching choices could be dangerous in the face of a strategic opponent but that switching might benefit them if he was supportive. In contrast, we see no similar pattern in the other versions of the problem, in which the protocol was fully defined so that host intent was logically irrelevant. In addition, a large share of subjects recognized that where the host offered money to switch, his intent was relevant and that, if he wished them to win, it was appropriate to switch.

The cue of an interested opponent is not available in the Three Prisoners Problem, since the setup does not have the structure of a contest. Nonetheless, if we interpret both problems as cases requiring the estimation of a pair of probabilities, they are formally very similar.[15] It is therefore significant that responses differ dramatically for the two problems. Table 1 shows that less than 20 percent of respondents choose to switch from their original choice when first facing the Monty Hall Dilemma (see also Granberg and Brown, 1995; Friedman, 1998), whereas in the Three Prisoners Problem, the proportion who claim the chance of a pardon remains at 1/3 for Dick after the warden names Harry is nearly 50 percent (Shimojo and Ichikawa, 1989).

The way in which respondents make use of the cue of the interested opponent is suggestive of a general approach to solving complex problems. Armed with knowledge of the problem's formal structure, we might suppose that a respondent would choose to switch according to the relative probabilities of the two choices. Those who place equal probabilities on the prize being behind doors 1 and 2 would presumably be equally likely to stick with door 1 as to switch to door 2. Yet, of those who say the odds are equal, a large majority choose to stick with their original choice when no side offer of money is available (Table 7), showing that subjects are not merely first calculating a probability and then making a choice in accord with it. We suggest that they are making the decision to stick in accord with a general rule that does not yield specific probabilities. Like the statisticians and others who have published versions of the Monty Hall Dilemma, naive subjects are not able to provide the correct probabilities of the game they are playing, yet the pattern of results implies that they are attending to relevant information.

We suggest that when asked to provide a quantitative answer underlying their decisions, subjects choose among solution strategies that provide such answers. Even though the Problem of the English Philosopher lacks the interested opponent, this class of problems may provide the only structure that many subjects can use to obtain probability estimates. Since such observationally similar problems are solved by assuming that the relative odds associated with any pair of outcomes remain unchanged when another option is omitted, subjects choose this solution.

John Maynard Keynes and Frank Knight independently developed a distinction between decisions in which the probabilities associated with various outcomes could be calculated, termed "risk," and those where probabilities could not be calculated, termed “uncertainty.” LeRoy and Singell (1987) attempted to explain the distinction between risk and uncertainty in terms of conventional decision theory. Risk, they suggested, involved circumstances in which probabilities could be estimated independent of a decision maker's action, while uncertainty involved circumstances where the very choice by the decision maker would interact with the probabilities of various outcomes.[16] Viewed in these terms, play against an opponent involves uncertainty. If the Monty Hall Dilemma is viewed as involving such play, this may, in part, explain the difficulty that both statistical professionals and the lay public display in their attempts to estimate appropriate probabilities. Subjects facing the Monty Hall Standard or Monty Hall Random problem act as they would in the face of uncertainty because they are unable to interpret the significance of the host's protocol, although, formally, they face risk. The disjuncture between subjects' probability estimates and their decisions suggests that while people have learned to play such games, they usually have little facility in calculating probabilities.

It is common for analyses of how people solve problems to distinguish between "cues," which are logically irrelevant to the problem, and elements that are formally necessary to obtain a solution. In contrast to this dichotomy, we believe that individual behaviors are best understood as evaluating information in terms of its success in solving previously encountered problems. Even when a particular item of information is not logically necessary for a solution, if it is a valuable aid in solving problems that are normally faced, it will be used. A particularly stark example in which social cues influence how people make logical judgments is provided in Gigerenzer and Hug (1992). They show that subject performance is enhanced dramatically if subjects are provided with a social context that supports the logical structure of a puzzle.

While such cues are generally valuable, when subjects face a problem that is outside their usual experience, cues they apply may not yield a narrowly correct solution. In the Monty Hall Dilemma, despite the fact that subjects routinely give an incorrect answer, the analysis here shows that they attend to elements of the problem that are relevant in related problems.

FOOTNOTES

1. This possibility was commented on by Monty Hall himself (Tierney, 1991), and in Engel and Venetoulias (1991). Nickerson (1996) provides a particularly illuminating discussion of this issue. See also Friedman (1998).

2. In the notation above, this means that $p_{ii}=p_{i1}=0$ for $i=1,2,3$, where p_{ii} is the chance that the warden reveals that individual i is pardoned, and p_{i1} is the chance he reveals the fate of Dick when individual i is pardoned.

3. In the notation of the previous section, this allows for the possibility that $p_{12} \neq p_{13}$. See references cited in Ichikawa and Takeichi (1990).

4. Shimojo and Ichikawa (1989) showed that where the initial chance of a pardon differed for Dick and Tom, such subjects almost always provided an answer that conserved the ratio of initial probabilities, suggesting use of a somewhat more sophisticated "constant ratio belief."

5. Although Nickerson (1996) emphasized the importance of the underlying assumptions in his analysis of the Monty Hall Dilemma, his analysis of the Three Prisoners Problem assumed without any discussion that the warden always agrees to the prisoner's request.

6. For example, recent texts in game theory (Rasmusen, 1994) and statistics (McClare, Dietrich & Sincinch, 1997), and a recent biography of mathematician Paul Erdös (Schechter, 1998) presented the defective version of the puzzle. The National Public Radio syndicated show CarTalk also presented a defective version of the puzzle in three separate shows broadcast in October 1997. Like earlier writers, in each case, those presenting the puzzle assumed they had fully specified the host's protocol, indicating that switching doors doubled the chance of winning.

7. In a sample of more than 1000 letters sent by readers in response to vos Savant (1990a), in which most letter writers argued that switching conferred no advantage, we found very few who

correctly and completely recognized the role of the host's protocol. Granberg and Brown (1995) found that subjects' response to the Monty Hall Dilemma did not depend on whether they were informed that the host had prior knowledge about the position of the automobile.

8. Our statements about the game are based on observing a dozen videotapes of "Let's Make a Deal," which were made during the 1990s when the show was rerun. See also Friedman (1998) for comment on the actual protocol followed by the host.

9. Friedman (1998) found that increasing subjects' payments for correct answers slightly *reduced* the chance that they would switch from an original choice, even where they played the game repeatedly with feedback about whether their answers were correct. Granberg and Brown (1995) observed approximately the same low level of switching in studies with no monetary incentives as in experiments where subjects could win a monetary reward for correct answers.

10. It may be argued that we have not specified the protocol in sufficient detail to eliminate the possibility that the host favors either door 2 or door 3 if neither has the automobile. However, the problem does specify that the subject has no way to distinguish among the three doors, which implies that she has no information that would allow her to distinguish among such host protocols, so that, by symmetry, the chance of winning by switching is 2/3. If the subject can distinguish among doors, the chance of winning by switching must be between 1/2 and 1, so switching is never an inferior choice.

11. Our reference here is to subjects' responses when they first encountered the puzzle, prior to any learning.

12. Several *ad hoc* explanations may be proposed for the position effect. For all respondents, the second problem differs only from the first in that host intent is altered. Subjects may have decided to "diversify" their answers, or they may have been cued to the possibility that answers

should be different in different versions ("Why would there be multiple versions of the same easy problem?"). We see no good way to distinguish these and other plausible explanations, given the design of the present study.

13. The prediction in the Negative Intent case is based on the assumption that the host does not know that the contestant is aware of his incentives. In contrast, if both host and contestant have full knowledge, the Nash equilibrium implies that the offer of money (if it ever occurs) will provide no useful information. Where the host wishes to increase the contestant's chance of winning, the Nash equilibrium implies that the host's offer will invariably provide information.

14. The exception is that in this sample of questionnaires the Money to Switch condition is confounded with position 4 effects. Since estimated effects of positions 2 and 3 are essentially identical in models reported above, we obtain identification by assuming that there are no substantial differences in effect for positions 2-4.

15. Whereas the warden may have unspecified interests that determine what information he provides (the desire not to transmit bad news, the "mum" effect), in contrast to the Monty Hall Dilemma, the prisoner does not have the opportunity to take any action. We noted above that the Three Prisoners Problem implies that $p_{ii}=p_{i1}=0$, for $i=1, 2, 3$, restrictions that are not imposed by the narrative of the Monty Hall Dilemma.

16. This latter interaction is what economists call "moral hazard." An example of moral hazard would be the case where a firm provides fire insurance, with the result that those who buy insurance take actions that increase the chance of a fire.

APPENDIX A: DERIVING NASH EQUILIBRIA

The chance that the prize (an automobile) is behind door *i* is p_i. We will assume p_1=p_2=p_3=1/3 in what follows. The host's protocol is defined by the p_{ij}, indicating the probability that the host opens door *j* when the prize is behind door *i*. We will assume that if the host opens a door, the contestant has the option to switch to any door, including the one that has been opened. We specify p_{i0} as the probability that the host opens no door, in which case the contestant must stick to her original choice. Throughout, we will assume that the contestant chooses door 1 initially. This structure allows for the possibility that the host may reveal the contents of the door chosen by the contestant and that he may reveal which door actually contains the automobile, possibilities not normally considered. In the experiments we report, the problem we label Monty Hall Ambiguous corresponds formally to this structure. The other problems we consider (labeled Monty Hall Standard and Monty Hall Random) formally specify the host protocol so there is little latitude for choice.

The contestant's strategy is defined by her decision of whether to stick with door 1 if a door is opened, and, if so, which door to switch to. Let q_{Gj}^k be the probability that the contestant ultimately chooses door *k* when a goat is revealed to be behind door *j*, with q_{Aj}^k similarly defined for the case where an automobile is revealed. For example, q_{G2}^1 is the chance that the contestant sticks with door 1 when a goat is revealed to be behind door 2, and q_{G2}^3 is the chance that she switches to door 3.

Table A-1 provides the probability for each path by which the contestant can win. For example, the first row lists the probability that the contestant wins because the automobile is behind door 1 and the host chooses to open no door ($p_1 p_{10}$). The second row indicates another path by which the contestant wins when the automobile is behind door 1, in which the host opens door 1 to reveal the automobile, and the contestant chooses to stick ($p_1 p_{11} q_{A1}^1$). The event described in the narrative, in which the host opens door 3 to reveal a goat, is indicated by row 4 or row 7.

The probability that the contestant wins is then given by the sum of the probabilities associated with all winning paths. Substituting p_1=p_2=p_3=1/3, and p_{10}=1 - p_{11} - p_{12} - p_{13}, and rearranging, this probability can be written as

$$P = \frac{1}{3}\Big[1 - p_{11}(1 - q_{A1}^1) + p_{21}q_{G1}^2 + p_{22}q_{A2}^2 + p_{31}q_{G1}^3 + p_{33}q_{A3}^3 - p_{12}q_{G2}^2 - p_{13}q_{G3}^3 \quad \textbf{(A-1)}$$

$$(p_{32} - p_{12})q_{G2}^3 + (p_{23} - p_{13})q_{G3}^2\Big]$$

Opposing Interests

The case of greatest interest is one in which the host and the contestant have opposing interests. In this case, the Nash equilibria include of all possible combinations of parameters for which the values p_{ij} minimize Equation A-1 while the values of q_{Gj}^k , q_{Aj}^k maximize it. We will also assume that the host and contestant act rationally given information they have at the point in the game where they make their decisions, which further limits possible equilibria (i.e., we consider “perfect” Nash equilibria).

Proposition 1: If the host opens a door to reveal an automobile, the contestant chooses that door, i.e., q_{A1}^1=q_{A2}^2=q_{A3}^3=1. The host never opens door 2 or door 3 to reveal an automobile (p_{22}=p_{33}=0).

Proof: The contestant's choice follows immediately from perfection, since choosing the automobile is strictly better than any alternative if the automobile is revealed. Given that the coefficients on p_{22} and p_{33} in Equation A-1 are positive, the host minimizes the contestant's chance of winning by choosing these to be zero.

Proposition 2: If the host were to open door 1 to reveal a goat, the contestant would choose to switch doors, i.e., $q_{G1}^{1}=0$. However, the host never opens door 1 to reveal a goat, i.e., $p_{21}=p_{31}=0$.

Proof: The contestant's decision follows from perfection, since switching is strictly preferred to sticking with door 1. We next show that any choice by the host other than $p_{21}=p_{31}=0$ leads to a contradiction.

a) If $p_{21}>p_{31}$ then the contestant chooses $q_{G1}^{2}=1$ and $q_{G1}^{3}=0$. But, in this case, the best response by the host implies $p_{21}=0$, which contradicts $p_{21}>p_{31}$. It can be shown analogously that $p_{21}<p_{31}$ is not possible, so it must be true that $p_{21}=p_{31}$.

b) If we assume that $p_{21}=p_{31}>0$, then it follows that the contestant's best response implies $q_{G1}^{2}+q_{G1}^{3}=1$. But in this case, the host's choice of $p_{21}=p_{31}>0$ is not optimal. It therefore must be the case that $p_{21}=p_{31}=0$.

Proposition 3: The probability that the host opens door 2 or door 3 to reveal a goat must be at least as high when the automobile is behind door 1 as when it is behind an unchosen door. This means that the host never provides useful information by opening a door, or, formally, $p_{12}\geq p_{32}$ and $p_{13}\geq p_{23}$.

Proof: If we assume the opposite, so that $p_{12}<p_{32}$, the contestant then chooses $q_{G2}^{3}=1$. But, in response to this, the host should choose $p_{12}=1$, implying a contradiction with $p_{12}<p_{32}$. Hence, it

must be the case that $p_{12} \geq p_{32}$ Similar reasoning shows that $p_{13} \geq p_{23}$.

Proposition 4: The contestant never switches doors when a goat is revealed behind door 2 or door 3, i.e. $q_{G2}^1 = q_{G3}^1 = 1$, and $q_{G2}^2 = q_{G2}^3 = q_{G3}^2 = q_{G3}^3 = 0$.

Proof: a) Contrary to the proposition, assume that $q_{G2}^1 < q_{G3}^1$. In response to this, the host's best choice is $p_{12}=1$ and $p_{13}=0$. (To see this, note that the contestant's chance of winning can be written so that the coefficient of p_{12} is (q_{G2}^1-1) and the coefficient of p_{13} is (q_{G3}^1-1).) In response to $p_{12}=1$, the contestant should choose $q_{G2}^1=1$. Hence, the assumption $q_{G2}^1 < q_{G3}^1$ cannot be true.

b) Reasoning analogous to that in (a) shows that $q_{G2}^1 > q_{G3}^1$ cannot be true.

c) Combining (a) and (b), it must be the case that $q_{G2}^1 = q_{G3}^1$. If $q_{G2}^1 = q_{G3}^1 < 1$, then the host's best choice implies $p_{12}+p_{13}=1$. But, in this case, $q_{G2}^1 = q_{G3}^1 < 1$ is not the contestant's best response. Hence, it must be the case that $q_{G2}^1 = q_{G3}^1 = 1$.

Proposition 5: The conditions specified in Propositions 1-4 together specify conditions that are both necessary and sufficient for a Nash perfect equilibrium.

Proof: Substitution into Equation A-1 in accord with the conditions in Propositions 1-4 shows that any choice consistent with these is optimal for both host and contestant.

There is substantial flexibility in possible equilibria, given these conditions. In equilibrium, the host may reveal the automobile when it appears in position 1, i.e., it is possible that $p_{11}>0$. The intuition for this result is that because, in equilibrium, the contestant always chooses to stick with her original choice, if her original choice is correct, it does not matter what the host reveals. Of course, the host's strategy must still create incentives for the contestant to stick with the original choice, so $p_{12} \geq p_{32}$ and $p_{13} \geq p_{23}$ (Proposition 3).

Among the many possible equilibria, the most obvious is one in which the host only opens a door and gives the contestant the opportunity to switch when she has chosen the correct door, i.e., p_{20}=1, p_{30}=1, $p_{12}+p_{13}$=1. In this case, the contestant always loses if she chooses to switch when given the opportunity, and so it is clear she never switches. However, the basic structure of the equilibrium is maintained even when the host sometimes offers the contestant the chance to switch when the automobile is behind an unchosen door. For example, if the host always gives the contestant the opportunity to switch when her initial choice is right, opening an unchosen door each time (e.g., $p_{12}=p_{13}$=1/2), and gives her the opportunity to switch in half the cases where her initial choice is wrong ($p_{32}=p_{30}$=1/2, $p_{23}=p_{20}$=1/2), this is also consistent with an equilibrium in which the contestant never switches.

Common Interests

What about the case where the host is attempting to help the contestant win the automobile? Equilibria are defined by parameter values that maximize the chance of winning. Let us remove from consideration the obvious equilibrium in which the host always opens up the door containing the automobile by constraining the choices such that p_{ii}=0, all i.

It is easy to see that a multitude of equilibria exist that assure the contestant will always choose the correct door. The essence of each is that actions by the host must provide sufficient information for the contestant to infer where the automobile is, and, if it is not behind door 1, must allow the contestant to switch doors.

If p_{10}<1, two types of equilibria exist: (1) $p_{12}=1-p_{10}$, $p_{23}=p_{31}$=1, and $q_{G1}^3=q_{G2}^1=q_{G3}^2$=1; and

(2) p_{13}=1-p_{10}, p_{21}=p_{32}=1, and $q_{G1}{}^{2}$=$q_{G2}{}^{3}$=$q_{G3}{}^{1}$=1. Hence, in both classes of equilibria, the contestant switches from the original choice when an unchosen door is opened at least some of the time. The equilibria in which p_{10}<1 are asymmetrical with respect to doors 2 and 3, so they do not seem natural in a context in which contestants are told that there is no way for them to distinguish between doors.

The most obvious equilibria are those in which the host does not open a door when the contestant has made the correct choice, i.e., those for which p_{10}=1. These equilibria fall into two classes: (1) p_{21}+p_{23}=1, p_{32}=1 and $q_{G1}{}^{2}$=$q_{G2}{}^{3}$=$q_{G3}{}^{2}$=1; and (2) p_{31}+p_{32}=1, p_{23}=1 and $q_{G1}{}^{3}$=$q_{G2}{}^{3}$=$q_{G3}{}^{2}$=1. In all these cases, the contestant always switches when given the opportunity. These equilibria include the one in which p_{23}=p_{32}=1, implying that the host reveals an unchosen alternative with a goat only when door 1 does not contain the automobile. If we restrict attention to equilibria for which the host never opens door 1, it is unique.

In short, in the most "natural" equilibria in which the contestant and host both want the contestant to win, the contestant switches when given the opportunity to do so. In other equilibria, the contestant will sometimes switch when given the chance.

APPENDIX B: EXPERIMENTAL DESIGN

Questionnaires were administered to groups of students in an introductory economics course who received extra course credit for participating. Oral instructions to the groups reinforced the written instructions in the introduction to the questionnaire. Group size varied from 15 to 125.

Each questionnaire contained nine pages. The first page provided an introduction, and the second page posed a question unrelated to this study. The next four pages contained variants of the Monty Hall Dilemma. For a given person, all questions were of the same basic type (Monty Hall Standard, Monty Hall Ambiguous, or Monty Hall Random) but differed according to host intent (Neutral Intent, Positive Intent, Negative Intent) and according to whether the host offered the contestant money to switch. (The exception was that, in some questionnaires, position 4 contained a variant of the Monty Hall Dilemma that we have not analyzed here.) In all, 15 variants of the Monty Hall Dilemma were included in the questionnaires, but their ordering varied across subjects. Table B-1 provides details regarding the ten questionnaire orderings used. Following the four problems, subjects were asked to explain their answers and provide information on their experiences with related problems. We reproduce below relevant material from the questionnaire, including one variant of the Monty Hall Dilemma.

Decision Making Questionnaire

This questionnaire will ask you to make a number of judgments involving hypothetical decisions. Read each question carefully, and provide the best answer that you can. It is important that you think through each question, but if you are unsure what the correct answer is, just make your best guess.

Please answer each of the questions in order. Once you have answered a question, do not return to that question unless the instructions tell you to do so, even if you think it would help you to answer a later question. Never change any answer on an earlier question even if you later believe you would have wanted to answer it differently.

Some of the questions will be quite similar, but no two questions are exactly alike. If you are not sure how a question differs from an earlier one, simply answer it as best you can based on your memory. Do not page back to the earlier question until the instructions indicate you should do so.

If you are unsure about the instructions at any point, please raise your hand.

[The next page contained a question not related to this study.]

[Each of the following four pages contained a variant of the Monty Hall Dilemma. As an example, the following question represents the condition: Monty Hall Standard-Negative Intent-Money to Switch. See the text for detail on variation across questions.]

121. A thoroughly honest game-show host has placed a car behind one of three doors. There is a goat behind each of the other doors. You have no prior knowledge that allows you to distinguish among the doors.

"First you point toward a door," he says. "Then I'll open one of the other doors to reveal a goat. After I've shown you the goat, you make your final choice whether to stick with your initial choice of doors, or to switch to the remaining door. You win whatever is behind that door."

Although the host will not give you any false information, his objective is to save the network money by reducing the likelihood that you win the car.

You begin by pointing to door number 1. The host shows you that door number 3 has a goat.

The host then says, "If you choose to switch from your original choice, I will give you an extra $100, whether or not you win the car."

What is your best estimate of your chance of winning, based on whether you choose to switch or stick? Give your answer as a percentage between 0% (no chance of winning) and 100% (certain to win).

Chance of winning if you stick with door 1 _____ %

Chance of winning if you switch to door 2 _____%

What would your final choice be? (Check one.)

____Stick with door 1 ____Switch to door 2

[The following question was on the page following the four variants of the Monty Hall Dilemma.]

100. We are interested in knowing how you came up with the answers to the decision problems on each of the previous four pages. At this point, page back to each of these questions, but do not change the responses you gave when you first answered them.

Provide a brief written explanation below saying how you estimated the probabilities you wrote down in the case of each of the four puzzles. If, after looking at the puzzles again, you now believe the answer you wrote down is wrong, try to explain the logic you used at the time. Then indicate below that you now think the answer is different from the one you gave, and why. (Do not change any answers on the pages above.)

If you used the same or similar reasoning in answering more than one of the questions, explain.

First puzzle

Second puzzle

[Approximately 1.5 in. provided for each of these answers.]

Third puzzle

Fourth puzzle

Have you heard or read previously about any of the four puzzles described above or any similar ones?

___Yes ____No

If yes, where was that? Please explain. [Approximately 2 in. provided for answer.]

Some people say that the problems listed above are like decisions they have often faced in actual life. Is there any real life decision like this you can think of? (Do not count statistics problems or puzzles.) [Approximately 3 in. provided for answer.]

[The questionnaire contained additional material unrelated to this study.]

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Table 1
Subject Response: Position 1, Neutral Intent

Problem Version	Percentage Switching	N
Monty Hall Standard	19%	54
Monty Hall Ambiguous	11%	53
Monty Hall Random	18%	56

Table 2: Logistic Model Predicting Switching: Selected Questions in Positions 1-3, Omitting Neutral Intent

Effect	Estimate
Intercept	-1.2652**
Monty Hall Standard	.1000
Monty Hall Ambiguous	-.2088
Monty Hall Random	.1088
Neutral Intent	.3808
Negative Intent	-.3555
Positive Intent	-.0353
Position 1	-.7874***
Position 2	.3712**
Position 3	.4162**

* $p \leq .1$ ** $p \leq .05$ *** $p \leq .01$
For each effect, estimated coefficients are constrained to sum to zero.

Table 3: Logistic Models Predicting Switching: Selected Questions in Positions 2-3 (Omitting Neutral Intent), With and Without Intent x Problem Type Interactions

	Model 1	Model 2
Effect	Estimate	Estimate
Intercept	-1.0601***	-1.0913***
Monty Hall Standard	.1908	.2262
Monty Hall Ambiguous	-.2858	-.3570*
Monty Hall Random	-.0878	.1308
Negative Intent	-.1447	.0223
Positive Intent	.1447	-.0223
Monty Hall Ambiguous		
Negative Intent		-.6323**
Positive Intent		.6323**

* $p \leq .1$ ** $p \leq .05$ *** $p \leq .01$
For each effect, estimated coefficients are constrained to sum to zero.

Table 4: Subject Response: Selected Questions in Positions 2-4 (Omitting Neutral Intent)

	Percentage Switching (N)			
Problem Version	No Money		Money to Switch	
	Negative Intent	Positive Intent	Negative Intent	Positive Intent
Monty Hall Standard	32% (54)	28% (54)	56% (27)	78% (27)
Monty Hall Ambiguous	11% (53)	30% (53)	33% (27)	67% (27)
Monty Hall Random	27% (56)	29% (56)	46% (28)	79% (28)

Table 5: Logistic Model Predicting Switching: Questions Omitting Neutral Intent

	Model 1	Model 2
	All Questions	Selected Questions in Positions 2-4
Effect	Estimate	Estimate
Intercept	-.7492***	-.3259
Monty Hall Standard	.1832	.2455*
Monty Hall Ambiguous	-.3116**	-.3628*
Monty Hall Random	.1284	.1173
No Money	-.7167***	-.7529***
Money to Switch	.7167***	.7529***
No Money		
Negative Intent	-.0871	-.0351
Positive Intent	.0871	.0351
Money to Switch		
Negative Intent	-.5341***	-.5024***
Positive Intent	-.5341***	.5024***
Monty Hall Ambiguous[a]		
Negative Intent	-.2411	-.4161*
Positive Intent	.2411	.4161*
Position 1	-.7888***	
Position 2	.3724**	
Position 3 or 4	.4164**	

* $p \leq .1$ ** $p \leq .05$ *** $p \leq .01$

[a] These coefficients indicate the additional impact of intent.

For each effect, estimated coefficients are constrained to sum to zero.

Table 6: Subject Response to Selected Questions in Positions 2-4 (Omitting Neutral Intent)

	Difference in Reported Chance of Winning if Switch			
Problem Version	No Money		Money to Switch	
	Negative Intent	Positive Intent	Negative Intent	Positive Intent
Monty Hall Standard	.071 (54)	.045 (54)	-.086 (27)	.144 (27)
Monty Hall Ambiguous	-.070 (53)	.081 (53)	-.136 (27)	.230 (27)
Monty Hall Random	.007 (56)	.015 (56)	-.046 (28)	.230 (28)

Table 7: Subject Response to Selected Questions in Positions 2-4 (Omitting Neutral Intent) for Subjects Reporting Equal Chance of Winning

	Percentage Switching (N)			
Problem Version	No Money		Money to Switch	
	Negative Intent	Positive Intent	Negative Intent	Positive Intent
Monty Hall Standard	21% (38)	19% (43)	75% (16)	74% (19)
Monty Hall Ambiguous	8% (40)	15% (39)	36% (14)	53% (15)
Monty Hall Random	18% (44)	21% (46)	50% (10)	80% (15)

Table 8: Subject Response to Open-End Questions for Problems in Positions 2-3 (Omitting Neutral Intent)

Problem Version	Subject Statement About Whether Host Intent Influences Probability Estimates			
	Host Intent Matters	Host Intent Does Not Matter	No Reference to Host Intent	N
Monty Hall Standard	16.7%	24.1%	59.2%	108
Monty Hall Ambiguous	26.4%	10.4%	63.2%	106
Monty Hall Random	25.0%	22.3%	52.7%	112

Table A-1
Probabilities for Paths in which the Contestant Wins

Path	Door with Auto	Door Opened	Stick/Switch	Probability contestant wins
1.	1	none	(Stick)	$p_1 p_{10}$
2.	1	1	Stick	$p_1 p_{11} q_{A1}^1$
3.	1	2	Stick	$p_1 p_{12} q_{G2}^1$
4.	1	3	Stick	$p_1 p_{13} q_{G3}^1$
5.	2	1	Switch to 2	$p_2 p_{21} q_{G1}^2$
6.	2	2	Switch to 2	$p_2 p_{22} q_{A2}^2$
7.	2	3	Switch to 2	$p_2 p_{23} q_{G3}^2$
8.	3	1	Switch to 3	$p_3 p_{31} q_{G1}^3$
9.	3	2	Switch to 3	$p_3 p_{32} q_{G2}^3$
10.	3	3	Switch to 3	$p_3 p_{33} q_{A3}^3$

Table B-1
Structure of Questionnaire

		Position 1	Position 2	Position 3	Position 4
Version	Basic Question Type	Intent/ Money to Switch?	Intent/ Money to Switch?	Intent/ Money to Switch?	Intent/ Money to Switch?
1.	Monty Hall Standard	Neutral/ No Money	Positive/ No Money	Negative/ No Money	Negative/ Money
2.	Monty Hall Standard	Neutral/ No Money	Negative/ No Money	Positive/ No Money	Negative/ Money
3.	Monty Hall Standard	Positive/ No Money	Negative/ No Money	Negative/ Money	(Unrelated Question)
4.	Monty Hall Standard	Negative/ No Money	Positive/ No Money	Positive/ Money	(Unrelated Question)
5.	Monty Hall Ambiguous	Neutral/ No Money	Positive/ No Money	Negative/ No Money	Negative/ Money
6.	Monty Hall Ambiguous	Neutral/ No Money	Negative/ No Money	Positive/ No Money	Negative/ Money
7.	Monty Hall Ambiguous	Positive/ No Money	Negative/ No Money	Negative/ Money	(Unrelated Question)
8.	Monty Hall Ambiguous	Negative/ No Money	Positive/ No Money	Positive/ Money	(Unrelated Question)
9.	Monty Hall Random	Neutral/ No Money	Positive/ No Money	Negative/ No Money	Negative/ Money
10.	Monty Hall Random	Neutral/ No Money	Negative/ No Money	Positive/ No Money	Negative/ Money