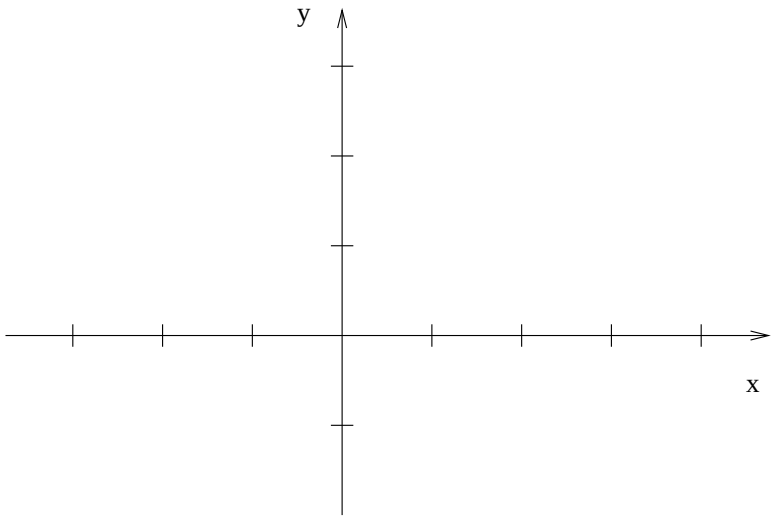
MAT120 Test 1 (Spring 2003): Functions, Limits, Derivatives

Name:

Directions: All problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. Consider the function f defined by

$$f(x) = \begin{cases} 1 & x < 1 \\ (x-1)^2 + 1 & x \geq 1 \end{cases}$$



1. Carefully graph f on the axes above. (3 pts)
2. Find the limits – provided they exist. Show your work! (2 pts)

$$\lim_{x \to 1^+} f(x)$$

$$\lim_{x \to 1} f(x)$$

3. Is f continuous at $x = 1$? Explain! (2 pts)

4. Is f differentiable at $x = 1$? Explain! (3 pts)

Problem 2. Consider the functions $f(x) = \sin(x)$ and $g(x) = \frac{1}{x+1}$.

1. What are the domains and ranges of the two functions?

2. Write formulas for $f \circ g$ and $g \circ f$ (make sure to distinguish which is which!).

3. Find the domains of $f \circ g$ and $g \circ f$?

4. Evaluate $f \circ g$ and $g \circ f$ at $x = 0$.

Problem 3. Give an example of each of the following, or explain why it can't happen:

1. A function continuous at a point, but not differentiable there.

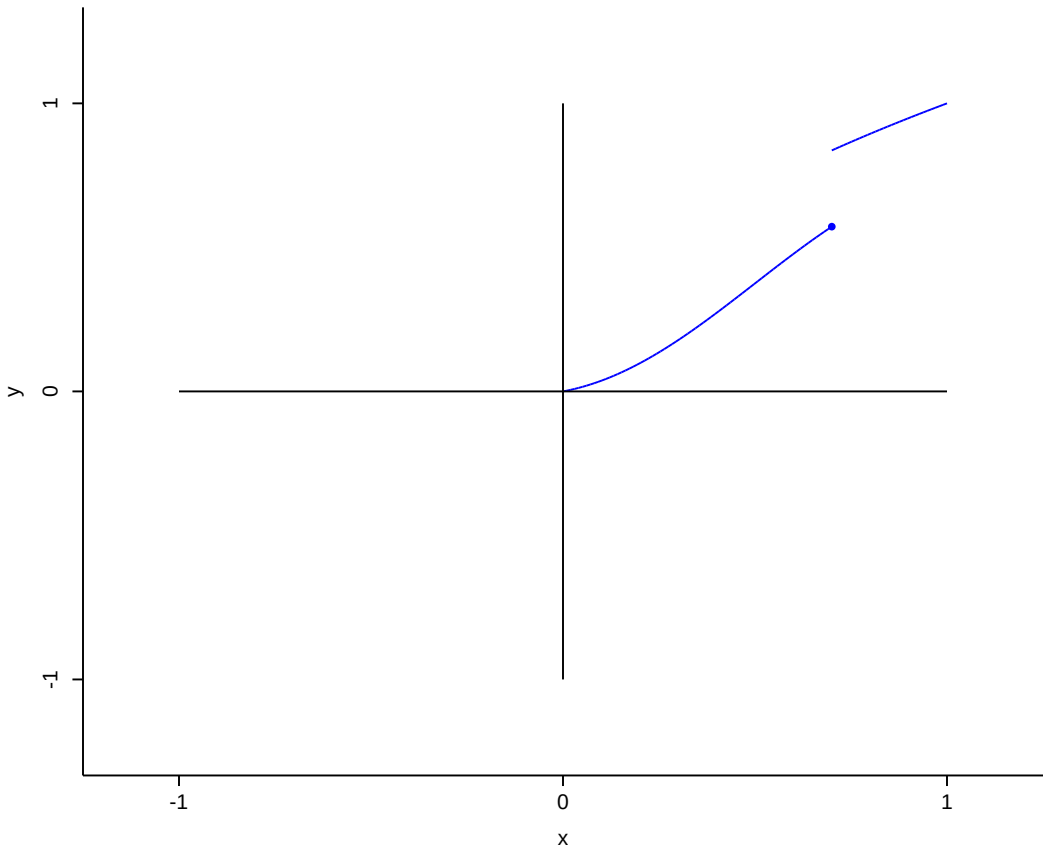
2. A function differentiable at a point, but not continuous there.

3. An even function that has an inverse.

4. A function f such that $f(2) = -2$ and $f(2) = 2$.

5. A function that fails to be differentiable in three qualitatively different ways.

Problem 4. Consider the function f defined on [0,1], as given the following graph (the dot at $x = .7$ means that the function is defined and takes that value there):



1. Carefully create three additional functions based on f, and add them to the plot above: (7.5pts)
   (a) An even function
   (b) An odd function
   (c) The inverse function (if it exists!)

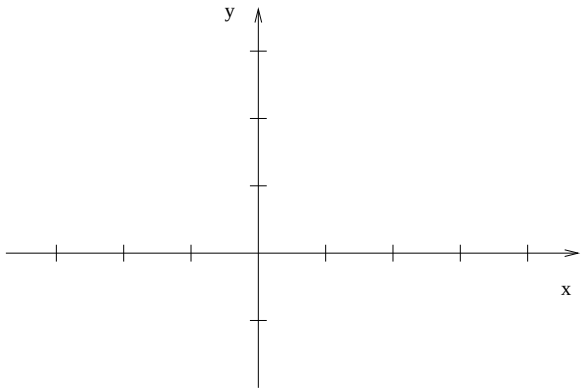
   Carefully label each piece corresponding to the three functions above in the graph.
2. Assuming that f's slope tends to zero as $x \to 0^+$, discuss the continuity and differentiability of the three functions listed above. (2.5pts)

Problem 5. A student is making steady progress in calculus class, as determined by weekly quiz scores given in the following table (out of 10 points):

Table 1: Quiz scores

Week	1	2	3	4	5	6	7
score (out of 10)	1	2	4	6	7	7	8

1. Graph the data, and draw a reasonable continuous function interpolating the data points. (4pts)



2. Estimate the instantaneous rate of change at week 2 of quiz score in two different ways: (4pts)

   (a) Using an appropriate average rate of change, and

   (b) Using an appropriate mathematical model.

3. If the trend of the data should continue, how well do you expect the rate of change of f at week 2 to represent the rate of change of f at week 8? Explain. (2pts)

Problem 6. Evaluate the limit

$$\lim_{x \to 3} \frac{x + \sqrt{x+6}}{|x+1|}$$

Citing appropriate limit laws, carefully explain how to find the limit (and then find it!).

Problem 7. Use the definition of the derivative at a point to find $g'(-1)$, where

$$g(x) = \frac{1}{\sqrt{x+2}}$$

Then find the equation of the tangent line to the graph of g at $x = -1$; finally use the tangent line to enhance a graph of g on the axes below.