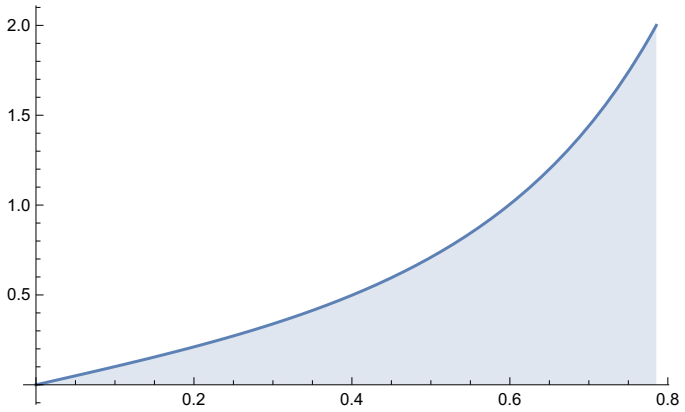
MAT229 Test 2 (Spring 2019): Integration techniques

Name:

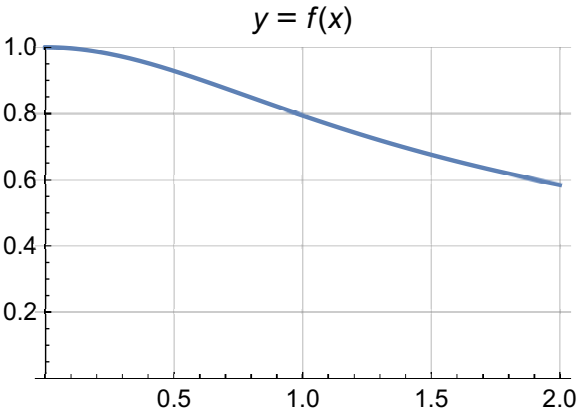
Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1 (10 pts): Compute the volume of the object obtained by rotating the function given by $f(x) = \tan(x)\sec^2(x)$ about the x-axis for $0 \leq x \leq \dfrac{\pi}{4}$.



Problem 2 (25 pts): Consider the definite integral $I = \int_0^2 \frac{1}{\sqrt[3]{x^2+1}} dx$ The integrand doesn't have a "nice" antiderivative, so the integral can only be approximated numerically ($I \approx 1.5980$).

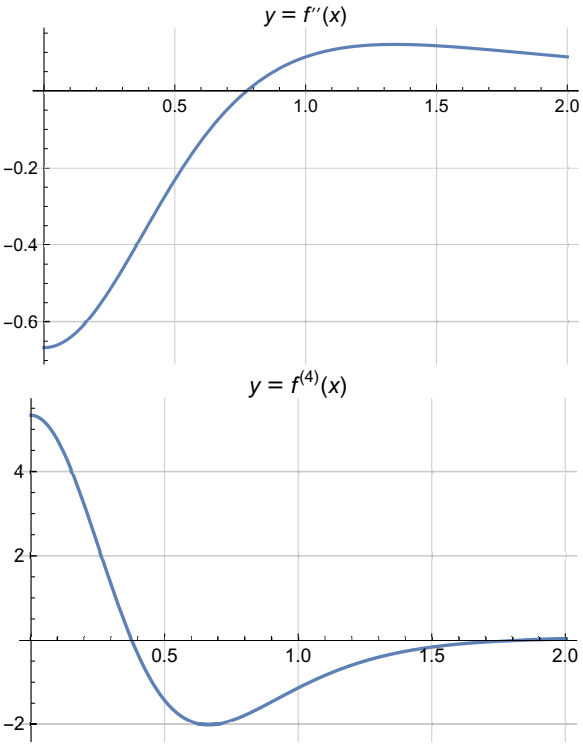
a. (12 pts) Your turn to approximate it: approximate this integral using left endpoint, right endpoint, midpoint and trapezoidal rules, with $n = 2$. Do the resulting errors make sense, given the figure?



method	estimate	error (estimate - 1.5980)
LRR		
RRR		
trap		
mid		

b. (4 pts) Derive a Simpson's estimate from part a. What is its "n" (how many rectangles used)?

c. **Problem 2**, cont. (9 pts) Using an appropriate error bound, what is the biggest error possible in midpoint, trapezoidal, and Simpson's estimates? Do your estimates and errors agree with your expectations? You may use the following graphs in your analysis:



method	$\|error\|$ bound	actual $\|error\|$
trap		
mid		
simp		

Problem 3 (30 pts): (10 pts each) Apply appropriate techniques of integration to evaluate each integral:

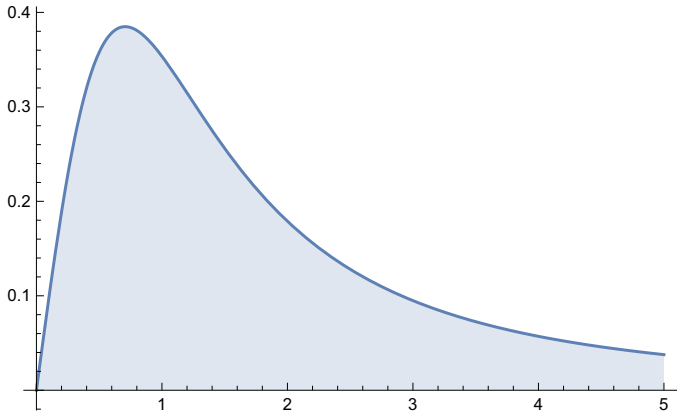
a. $I = \int xe^x dx$

b. $J = \int_0^1 \ln(x)dx$

c. $K = \int \frac{2x^2 + 5}{x^2 - 5x}dx$

Problem 4 (10 pts): What is the partial fraction decomposition of $\dfrac{x-2}{(x^2+1)(x+1)^2}$?

Problem 5 (10 pts): Consider the infinite strip given by $0 \leq y \leq \dfrac{x}{(x^2+1)^{3/2}}$ for $0 \leq x < \infty$ (part of which is shown below). Determine if it has finite area or not. If it does have finite area, find that area; if it doesn't have finite area, explain why not.



Problem 6 (15 pts): Compute the integral $I = \int_0^3 \frac{x^3 dx}{\sqrt{25 - x^2}}$ by trig-substitution.