Number Theory Section Summary: 2.1

The Division Algorithm

"...the foundation stone upon which our whole development rests." (p. 17)

1. **Theorems**

Division Algorithm: Given integers a and b, with $b > 0$, there exist unique integers q and r satisfying

$$a = qb + r$$

with $0 \leq r < b$. q is called the **quotient**, and r is called the **remainder**.

If $a > 0$ as well, then this is an obvious extension of the Archimedean property: if any positive b can be added to itself enough times to exceed any positive a, then clearly there will come a point at which $qb \leq a$ and $(q+1)b > a$. r just represents the amount by which qb is short (if any!).

(Proof using well-ordering and contradiction.)

Corollary: Given integers a and b, with $b \neq 0$, there exist unique integers q and r satisfying

$$a = qb + r$$

with $0 \leq r < |b|$.

2. Notes

The author shows a couple of interesting properties immediately:

- $b = 2$ leads to the definition of even and odd numbers, as $2q$ or $2q + 1$.
- Furthermore, every square of an integer is of the form $4k$ or $4k+1$.
- $b = 4$ leads to the conclusion that every square of an odd is of the form $8k + 1$.

3. Summary

Burton comments that the focus will fall on the **applications** of the division algorithm: "...it allows us to prove assertions about all the integers by considering only a finite number of cases." (p. 19)