Section Summary: 14.1

a. Definitions

A **function f of two variables** is a rule that assigns to each ordered pairs of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the **domain** of f and its **range** is the set of values that f takes on D, that is, $\{f(x, y)|(x, y) \in D\}$. The variables x and y are the independent variables, whereas $z = f(x, y)$ is the dependent variable.

The **graph** of a function f of two variables is the set of all points (x, y, z) in $\Re^3$ such that $z = f(x, y)$ and (x,y) is in D.

A **function f of n variables** is a rule that assigns to each ordered n-tuple of real numbers $(x_1, x_2, \ldots, x_n)$ in a set D a unique real number denoted by $f(x_1, x_2, \ldots, x_n)$. The set D is the **domain** of f and its **range** is the set of values that f takes on D, that is, $\{f(x_1, x_2, \ldots, x_n)|(x_1, x_2, \ldots, x_n) \in D\}$.

We will take the preceding as our definitions of these multivariate functions, following the author of our textbook. You should realize, however, that functions could assign vectors, say, rather than numbers, or even functions. For example, "differentiation" is an operator (we use that name to distinguish it from functions on numbers), that takes functions as inputs, and produces functions as outputs.

In the univariate case, we have space curves, which are traced out in time (the trajectory of a planet, for example). The value of the function at a point (in time) is a location (and not a number), better represented perhaps as a vector. We can do ordinary calculus on these space curves, as described in chapter 13 of your text.

Similarly, if we have a temperature distribution in a room, for example (temperature as a function of three – or four – inputs, e.g. x, y, z, t), another function would be the **vector** direction of heat **flow**. That, too, would be a function of three – or four – inputs – but not in the sense of the definition above.

level curves of a function f of two variables are curves with equations $f(x, y) = k$, where k is a constant (in the range of f). More generally, **level surfaces** of a function f of n variables are surfaces with equations $f(x_1, x_2, \ldots, x_n) = k$, where k is a constant (in the range of f).

Imagine the level **surfaces** associated with a function of three variables, e.g. the temperature distribution in a room with a constant influx of heat from a window and a constant influx of cool air from an air conditioner.

f is a **linear** function if it is linear in each of its independent variables. Our author comments (and I concur strongly) that "...in much the same way that linear functions of one variable are important in single-variable calculus, we will see that linear functions of two variables play a central role in multivariable calculus." (p. 905)

b. Theorems

None.

c. Properties/Tricks/Hints/Etc.

You need to learn to draw “fake 3-d”! Practice, practice, practice....

d. Summary

Most of the focus of this text will be on functions of two-variables, whose graphs are surfaces in three-dimensions. One of the primary problems is that of how to represent these surfaces in two-spaces (i.e. on the board, on your paper).

Functions are defined in four ways:

- verbally
- numerically
- algebraically
- visually

But we tend to work most in this course with functions defined algebraically.

We may find it useful to use level curves, or level surfaces, to help us visualize multivariate functions. In the case of functions of two variables, we obtain level curves, which comprise a topographic map of the function.