Section Summary: 14.8
Lagrange Multipliers

a. **Definitions**

b. **Theorems**

c. Properties/Tricks/Hints/Etc.

The following is the formulation for the Method of Lagrange Multipliers in the trivariate case. It works in general dimensions, of course.

Method of Lagrange Multipliers: To find the extrema of $f(x, y, z)$ sugject to the constraint $g(x, y, z) = k$ (assuming that extrema exist),

i. Find all values of x, y, z, and λ such that

$$\nabla f(x, y, z) = \lambda \nabla g(x, y, z)$$

and $g(x, y, z) = k$.

ii. Evaluate f at all points (x, y, z) are solutions to step 1: the largest is the max, whereas the smallest is the min.

An alternate formulation of Lagrange multipliers is that we seek extrema of a function

$$F(x, y, z, \lambda) = f(x, y, z) - \lambda(g(x, y, z) - k)$$

Differentiate with respect to x, y, z, and λ; what equations do you derive?

The **sign** of λ is not important: that is, we can't tell from the formulation whether λ is positive or negative. It's also not really important in the problem: we just want the two gradients to be aligned – they don't have to both be pointing in the same direction.

d. Summary

Lagrange multipliers are simply means of introducing constraints into an optimization problem. Perhaps the easiest way to think of this is to imagine two surfaces intersecting, and to ask what the largest value of the first function is upon the curve of intersection.