Section Summary: 15.1

Riemann sum: the sum

$$\sum_{i=1}^{n} f(x_i^*)\Delta x$$

named after Bernhard Riemann (1826-1866), a student of Gauss. Here x_i^* is a sample point for an interval, and Δx is a fixed characteristic chunk of length for each interval.

Generalized:

$$\int\int [f(x,y) + g(x,y)]dA \approx \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A$$

Here (x_{ij}^*, y_{ij}^*) is a sample point for a subregion, and ΔA is fixed characteristic chunk of area.

We are not confined to equal chunks, of course.

a. Definitions

$$\int\int [f(x,y) + g(x,y)]dA = \lim_{m,n\to\infty} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_i, y_j)\Delta A$$

Here (x_i, y_j) is the upper bounding values of each subregion.

b. Theorems

One theorem stated within the text is that all continuous functions are integrable on a finite region (as are all bounded functions).

c. Properties/Tricks/Hints/Etc.

Properties of Double Integrals:

$$\int\int [f(x,y) + g(x,y)]dA = \int\int f(x,y)dA + \int\int g(x,y)dA$$

$$\int\int cf(x,y)dA = c\int\int f(x,y)dA$$

These combine to say that integration is a **linear** process: the integral of a linear combination of functions is the linear combination of the integrals of the functions.

d. Summary

When defining the univariate integrals, the canonical focus is on **area**. In bivariate (and higher) cases, we simply add a dimension when dealing with bivariate functions, and focus on volumes. So how to generalize the integral? In the obvious ways!