Section Summary: 15.2
Iterated Integrals

a. Definitions

Partial integration: integrating with respect to one variable, while holding the other(s) constant. This is the analogue of partial derivatives.

An **iterated integral** is one carried out with respect to one direction (e.g. x), then with respect to the other (e.g. y).

b. Theorems

Fubini's theorem: If f is continuous on the rectangle $R = \{(x, y)|a \leq x \leq b, c \leq y \leq d\}$ then

$$\iint_R f(x, y)\mathrm{d}A = \int_a^b \int_c^d f(x, y)\mathrm{d}y\mathrm{d}x = \int_c^d \int_a^b f(x, y)\mathrm{d}x\mathrm{d}y$$

This is true even if f is merely bounded on R, f is discontinuous only on a finite number of smooth curves, and the iterated integrals exist.

c. Properties/Tricks/Hints/Etc.

An important special case is when you have a "separable" function: that is, one that can be written $f(x, y) = g(x)h(y)$. Then

$$\iint_R f(x, y)\mathrm{d}A = \int_a^b g(x)\mathrm{d}x \int_c^d h(y)\mathrm{d}y$$

d. Summary

It's going to turn out to be useful to do integration one variable at a time, especially when on rectangular regions. And there are special cases, such as separable functions, when multivariate integration reduces directly to the univariate case (with twice the work).