**Section Summary: 15.3**

Double Integrals Over General Regions

a. **Definitions**

**Type I Region**: $D = \{(x,y) \mid a \le x \le b, \quad g_1(x) \le y \le g_2(x)\}$
**Type II Region**: $D = \{(x,y) \mid h_1(y) \le x \le h_2(y), \quad c \le y \le d\}$

b. **Theorems**

If region $D$ is type I, then

$$\iint_D f(x,y)\mathrm{d}A = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\mathrm{d}y\mathrm{d}x$$

If region $D$ is type II, then

$$\iint_D f(x,y)\mathrm{d}A = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\mathrm{d}x\mathrm{d}y$$

Notice the switch in the order of integration: in Type I, it was $\mathrm{d}y\mathrm{d}x$ and for Type II it was $\mathrm{d}x\mathrm{d}y$. Those little differentials are tremendously important – be careful that you don't drop them (especially around me!).

(By the way, I sometimes drop them myself – just checking to see if you're paying attention!;)

c. **Properties/Tricks/Hints/Etc.**

The usual properties hold: integration is still linear, regions can be broken up into a disjoint union (no overlap; all parts together make up the whole) and be treated separately.

It's kind of late for the author to introduce property 10 (p. 1018):

$$\iint_D 1 \ \mathrm{d}A = A(D)$$

where $A(D)$ is the area of the region of integration. I hope that it's clear. But one could (and perhaps **should**) still think of it as a volume: it's just **a volume of unit height**.

Property 11 should also be obvious when one thinks in terms of volumes: if $m \leq f(x,y) \leq M$, then

$$m \cdot A(D) \leq \iint_D f(x,y) \mathrm{d}A \leq M \cdot A(D)$$

## d. Summary

Our author throws a few little insights into this section, but basically it seems pretty straightforward.