Section Summary: 15.4
Double Integrals in Polar Coordinates

a. Definitions

Converting between polar and Cartesian coordinates:

$$r^2 = x^2 + y^2 \qquad x = r\cos\theta \qquad y = r\sin\theta$$

Polar rectangle: $R = \{(r, \theta) \mid a \le r \le b,\ \alpha \le \theta \le \beta\}$

b. Theorems

Writing a rectangular integral as a polar integral:

$$\iint_R f(x, y)\mathrm{d}A = \int_\alpha^\beta \int_a^b f(r\cos(\theta), r\sin(\theta))r\mathrm{d}r\mathrm{d}\theta$$

Notice that $\mathrm{d}A = r\mathrm{d}r\mathrm{d}\theta$: don't forget the extra term of r (the dimensions don't work out otherwise, since $\mathrm{d}\theta$ is dimensionless – we need an area!

Similarly to the "more general" regions of section 15.3, we can consider "more general" regions of polar regions. See figure 7, p. 1024.

If f is continuous on a polar region of the form $D = \{(r, \theta) \mid \alpha \le \theta \le \beta, h_1(\theta) \le r \le h_2(\theta)\}$, then

$$\iint_R f(x, y)\mathrm{d}A = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} f(r\cos(\theta), r\sin(\theta))r\mathrm{d}r\mathrm{d}\theta$$

c. Properties/Tricks/Hints/Etc.

d. Summary

There are times when circles are the right kinds of objects to use to represent regions. In these times, it's often better to go to another kind of coordinate system. In this section we consider polar coordinates (or cylindrical, in three dimensions). Later on, we'll also consider spherical coordinates. Both of these systems are important especially in physics, or engineering, where we seem to love circles (with good reason, by the way – nature loves a sphere!).