Section Summary: 16.7
Triple integrals

a. **Definitions**

The **triple integral** of f over the box B is

$$\iiint_R f(x,y,z)\mathrm{d}V = \lim_{l,m,n\to\infty} \sum_{i=1}^{l}\sum_{j=1}^{m}\sum_{k=1}^{n} f(x^*_{ijk}, y^*_{ijk}, z^*_{ijk})\nabla V$$

b. **Theorems**

Fubini's theorem for triple integrals: If f is continuous on the box $B = [a,b]\times[c,d]\times[r,x]$, then

$$\iiint_R f(x,y,z)\mathrm{d}V = \int_r^s \int_c^d \int_a^b f(x,y,z)\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

and all other permutations of the integral are equivalent.

c. Properties/Tricks/Hints/Etc.

Similar to the "Type I" and "Type II" regions of bivariate integrals, we have special regions for trivariate integrals. Type I is one where x and y lie within a fixed domain D, and values of z lie between functions of x and y: $E = \{(x,y,z) \mid (x,y)\in D,\ u_1(x,y)\le z\le u_2(x,y)\}$.
Then

$$\iiint_R f(x,y,z)\mathrm{d}V = \int_r^s \int_c^d \left(\int_{u_1(x,y)}^{u_2(x,y)} f(x,y,z)\mathrm{d}x\mathrm{d}y\right)\mathrm{d}z$$

d. Summary There is no big news here: life just gets more complicated,

as we are required to consider more permutations of types of regions, etc. Hopefully a few examples will illuminate the situation.