Section Summary: 16.1
Vector Fields

a. Definitions

Let D be a set in $\Re^2$. A **vector field on** $\Re^2$ is a function $\mathbf{F}$ that assigns to each point (x, y) in D a two-dimensional vector $\mathbf{F}(x, y)$.

Let E be a set in $\Re^3$. A **vector field on** $\Re^3$ is a function $\mathbf{F}$ that assigns to each point (x, y, z) in E a three-dimensional vector $\mathbf{F}(x, y, z)$.

A **gradient field** is a field derived as the gradient of a scalar function f. Such a vector field $\mathbf{F}$ is called **conservative**, and f is called a **potential function** for $\mathbf{F}$.

b. Theorems

c. Properties/Tricks/Hints/Etc.

If we think of the point (x, y, z) as a vector $\mathbf{x}$, then we can think of vector functions as taking vectors to vectors:

$$\mathbf{x} \longrightarrow \mathbf{F}(\mathbf{x})$$

d. Summary

Vector fields are all around us: any phenomenon which associates with a point a vector can be represented using the concept of a vector field. The wind at every point; the temperature gradient in a room (an example of a gradient field, derived from the scalar temperature function at each point); the velocity of a spatially-defined epidemic (like my raccoon rabies problem).

The gradient fields will have special properties, as we will see, and are extremely important in physics. Gravitational and electrical fields are examples of these, and have the special property that they share of being the consequence of "inverse distance square" laws.