MAT360 Section Summary: 1.3
Algorithms and Convergence

1. Summary

There are three separate issues discussed in this section:

- algorithms,
- convergence and growth of errors, and
- order of error (big O).

An algorithm is a recipe for completing a task. As we've seen, algorithms giving the same answer from the purely mathematical standpoint may give radically different answers from a numerical perspective. So we want to make good choices when we create algorithms.

If an algorithm has the property that small changes in initial conditions produce small changes in the solution, then the algorithm is **stable**; otherwise it is **unstable**. Some algorithms are stable for a range of initial data, and they might be categorized as **conditionally stable**.

"Nice" errors (if there can be such a thing!) have the property that errors introduced at the outset grow linearly, i.e. as

$$E_n \approx CnE_0$$

where $C > 0$ is independent of n. If, on the other hand, the errors grow exponentially,

$$E_n \approx C^n E_0$$

where $C > 1$ is independent of n, then we're probably going to be in trouble before we'd like! The good news is that your bank account grows unstably, if you like!

2. Definitions

- **Definition 1.18** Suppose $\{\beta_n\}_{n=1}^{\infty}$ is a sequence which converges to zero, and $\{\alpha_n\}_{n=1}^{\infty}$ converges to a number α. If $\exists K > 0$ with

$$|\alpha_n - \alpha| \leq K|\beta_n|$$

for large n, then $\{\alpha_n\}_{n=1}^{\infty}$ converges to α with **rate of convergence** $O(\beta_n)$.

- **Definition 1.19**: Suppose that $\lim_{h\to 0} G(h) = 0$ and $\lim_{h\to 0} F(h) = L$. If $\exists K > 0$ /

$$|F(h) - L| \leq K|G(h)|,$$

for sufficiently small h, then $F(h) = L + O(G(h))$.

3. Properties/Tricks/Hints/Etc.

β_n is generally of the form of powers of $\frac{1}{n}$.