MAT360 Exam 2 (Spring 2017): Root-finding and Interpolation

Name:

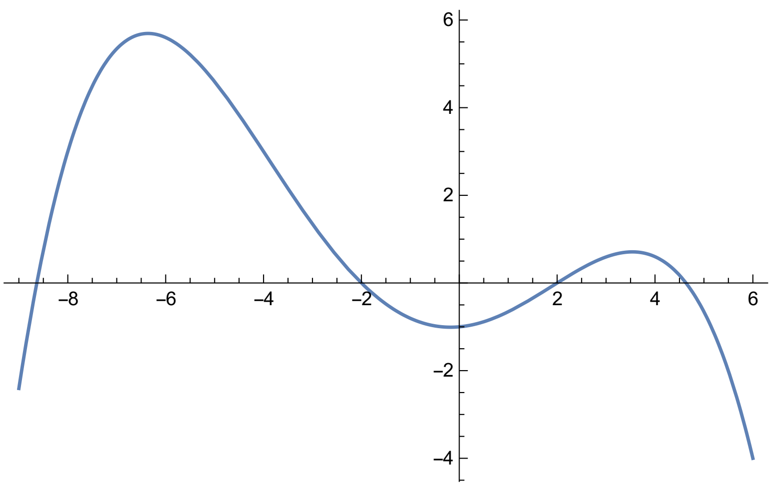
Directions: You must skip one problem – write "skip" prominently on it. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. (10 pts)

a. For the given function (a quartic, of form

$$p(x) = -1 + x/10 + (11x^2)/40 - x^3/40 - x^4/160$$

we have the following graph:



a. (4 pts) Use the figure to illustrate how Newton's method works starting from the guess $x = 0.5$. Include the calculation of the next iterate (you don't need to do it in general, just for $x = 0.5$), and you can use these values: $p(0.5) = -0.884766$, and $p'(0.5) = 0.353125$.

b. (2 pts) $p(x)$ is not expressed efficiently for calculation. How might you write it better?

c. (4 pts) The quadratic form of Newton relied on fitting the "tangent quadratic". Knowing that $p''(0.5) = 0.45625$, write the tangent quadratic at $x = 0.5$. (Hint: use Taylor's Theorem.)

Problem 2. (10 pts) We've studied how to compute linear splines and the errors we make in using them. Suppose that we use the "knots" $(x_i, y_i) = (i * h, \sin(x_i))$, where $h = 0.1$ and create a linear spline interpolator for $\sin(x)$. (Calculations below are in radians, of course).

a. (3 pts) Write an explicit formula for the piece of the linear spline $s(x)$ used to estimate $\sin(.55)$. What is the estimate for $\sin(.55)$, and how does it compare to the true value?

b. (3 pts) We can make some statements about the error we make in estimating $\sin(.55)$ using the linear spline, by considering properties of $\sin(x)$ in the neighborhood of $x = .55$. What can we say?

c. (3 pts) Bound the error we make in estimating $\sin(.55)$ using the linear spline.

Problem 3. (10 pts) Consider the three points

x_i	$f[x_i]$
$x_0 = 0$	1
$x_1 = 1$	2
$x_2 = 2$	-2

a. (3 pts) Complete the divided-difference table above.

b. (3 pts) Write the Newton interpolating polynomials (from constant to quadratic) obtained by successively adding the points in the order x_1, x_2, and x_0.

c. (3 pts) Write the interpolating polynomial in Lagrange form.

d. (1 pt) What is the difference between these polynomials?

Problem 4: (10 pts) Discuss the advantages and disadvantages of each of the following root finding methods. Mention order of convergence when you can, dangers, possibilities, mathematical underpinnings, etc. Show me that you understand each method.

Method	Advantages	Disadvantages
Newton's		
Secant		
Muller's		
Bisection		

Problem 5. (10 pts) We seek a root of

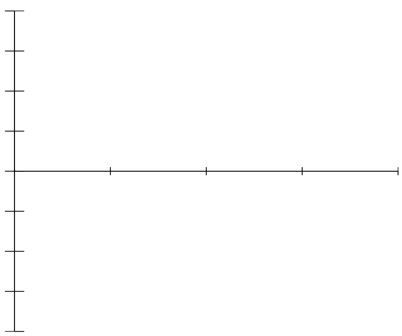
$$f(x) = x + \ln(x)$$

You consider two possibilities, both of them fixed-point methods: Newton's method, and the more straightforward method suggested by

$$f(x) = 0 \iff x = -\ln(x)$$

So define $g(x) = -\ln(x)$.

a. Sketch (roughly) the graphs of x and $-\ln(x)$ below, and conclude that there is a unique root:



b. Explain why $g(x)$ is a poor choice for a fixed-point iteration scheme.

c. Newton's method is also a fixed-point iteration scheme. What is Newton's fixed-point function to find roots of f?

Problem 6. (10 pts) Determine a low-degree polynomial approximation to

$$f(x) = \cos(x) - 1$$

for $|x| < 0.1$ having a relative error of less than or equal to 10^{-3} in magnitude. (Hint: Taylor series remainder term, and both numerator and denominator are easy to bound.)