MAT385 Test 1 (Fall 2016): 1.1-1.4; 2.1-2, 2.4

Name:

Directions: Problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1: (10 pts)

a. (6 pts) Prove the following with a truth table:

$$(P \to Q) \wedge (Q' \vee P) \to (Q' \wedge P)'$$

I'll get you started, and you finish it:

P	Q	$P \to Q$	$Q' \vee P$	
T				
T				
F				
F				

b. (4 pts) Suppose that P and Q above are given by

- P: It is raining.
- Q: The sky is grey.

Translate the propositional wff above into "common speech" (try not to make it too ugly), and explain why it makes sense (and hence can be proven!).

Problem 2: (10 pts) Use propositional logic to prove the wff in Problem 1. You may want to restate the given wff as an **equivalent** wff:

$$(P \to Q) \wedge (Q' \vee P) \to (Q' \wedge P)'$$

Problem 3: (10 pts) Every book is loved by some person. No person loves all books. "To Kill a Mockingbird" is a book. Therefore, someone does not love "To Kill a Mockingbird".

a. Express this argument as a predicate wff. Use

- B(x) - x is a book
- P(y) - y is a person
- L(x,y) - y loves x
- k - "To Kill a Mockingbird"

b. Is this argument valid or not? If valid, prove it; if not, explain.

Problem 4: (10 pts) Decide if the following wff is valid, and either give an interpretation in which it is false, or prove it:

$$(\exists x)\,(P(x) \to Q(x)) \wedge (\forall y)\,(Q(y) \to R(y)) \to ((\forall x)P(x) \to (\exists x)R(x))$$

Problem 5: (10 pts) Prove that the square of an odd integer n is $8k + 1$ for some integer k.

Problem 6: (10 pts) Use induction to prove the formula for the sum of the first n terms of a geometric sequence, for $n \geq 1$:

$$P(n): \quad a + ar + ar^2 + ... + ar^{n-1} = a\frac{1-r^n}{1-r}$$

Problem 7: (10 pts)

a. Give a proper recursive definition for the set of all palindromic strings (strings that read the same forward as backward) **having an even number of 0s**. The alphabet is binary digits, 0 and 1 (and you'll need the empty string, too – designated λ).

   You might start by listing some valid palindromic strings: your definition should produce each of these.

b. A male bee has only one parent (a female – the queen); a female bee has two parents (the queen and her male mate).

   i. Draw a family tree diagram showing the ancestors of a male bee (call it M), for five generations starting from generation 1 – the first bee. Use F for a female, and M for a male.

   ii. How many bees occur in each generation? Identify the sequence of counts of bees $C(n)$, where n is the n^{th} generation, and so write a recurrence relation for the counts.