Name:

Directions:

- All problems are equally weighted (10 points each).
- Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning).
- Indicate clearly your answer to each problem (e.g., put a box around it).
- **Good luck!**

Problem 1. Suppose that

1. Fibonacci Nim is to be played starting with 20 sticks,
2. you start, and
3. your opponent always pulls two sticks.

Write the sequence of winning moves you will use to defeat player two.

Problem 2. Each box of Animal Crackers contains 4 servings with exactly 12 crackers in a serving. There are exactly 15 different shapes made.

1. Explain why there always two crackers of the same shape in each box.
2. In fact, one is guaranteed to have at least how many of the most abundantly represented shape in each box? (Remember to show your work, too!)

Problem 3. Demonstrate how to put two segments of different lengths into one-to-one correspondence, so as to indicate that both sets have the same cardinality.

Problem 4. Two Cups, Four Ounces. You have two measuring cups: one holds exactly 5 ounces of water and the other holds exactly 3 ounces of water. There are no markings on the cups and you are not able to mark the cups at all. You are given a huge bucket of water. Is it possible to measure and place exactly 4 ounces of water into the big cup? If so, carefully explain your method; if not, carefully explain why not.

Problem 5. True or False?

1. [( )] If the number M is an irrational number, then 1/M must be an irrational number as well.
2. [( )] If a real number has an infinitely long decimal expansion, then it must be an irrational number.

nalities.

4. ( ) In "Let's Make a Deal", Warren Piece (as the raisin) could either stick with the initial choice of doors, or switch, once Monty Hall had shown the mule behind one of the doors. In general, Warren would be foolish to switch.

5. ( ) Cantor's diagonalization proof demonstrates that the natural and rational numbers can be put into a one-to-one correspondence.

Problem 6. Show that $\sqrt{5}$ is irrational.

Problem 7. Prove that $.5\overline{9} = .60$.

Problem 8. Demonstrate that the natural numbers and the rational numbers can be put into one-to-one correspondence.