MAT115 Fall 2003: Test 1

Name:

Directions: All problems are equally weighted. You should do 4 of 5, writing "skip" clearly on the one you do not wish me to grade. You **must** skip one!

Show your work: be as thorough as you can be. Simple answers without justification will likely result in few points. **Good luck!**

Problem 1. You're working the desk of the Hotel Cardinality (rooms numbered 1, 2, 3, ...) on a night when the hotel is empty. A bus pulls up outside. It's long and slim, and looks a lot like the real number line (so it's really long, and really slim!). On this sleek bus there is a seat for every real number, and the bus is full. Of course, everyone wants a room.

- Briefly tell the newcomers of your experience with large busses.

- In the end, what can you do for the folks on this bus?

Problem 2. Find a rational number on the real number line strictly between $\sqrt{2}$ and 2, and then find an irrational number strictly between the two.

From this example, play Devil's advocate and briefly argue that the rational numbers and the irrational numbers should have the same size as infinite sets!

Problem 3. Play a game (to win!) of Fibonacci Nim against an opponent P1 whose strategy is to always take away the largest prime p where $p \leq 1/4$ of the number of sticks remaining, or the maximum allowable (if it is against the rules to take that prime). You are player two, or P2, and the game begins with 54 sticks. Show all moves.

Problem 4. Someone offers you a million dollars under one condition: you have to carry it off in $1 bills, in a single load under your own power. No tricks: you must literally carry it all off at once, on your back, on your head, whatever – but under your own power. Do you take the offer? Details please!

Problem 5. Without using the fact that there is an infinite supply of distinct primes, demonstrate that there is a prime number bigger than one billion.