MAT120 Test 2 (Spring 2003): Derivatives

(sections 3.2-3.6, 3.8, 3.9, and 4.1-4.3)

Name:

Directions: All problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

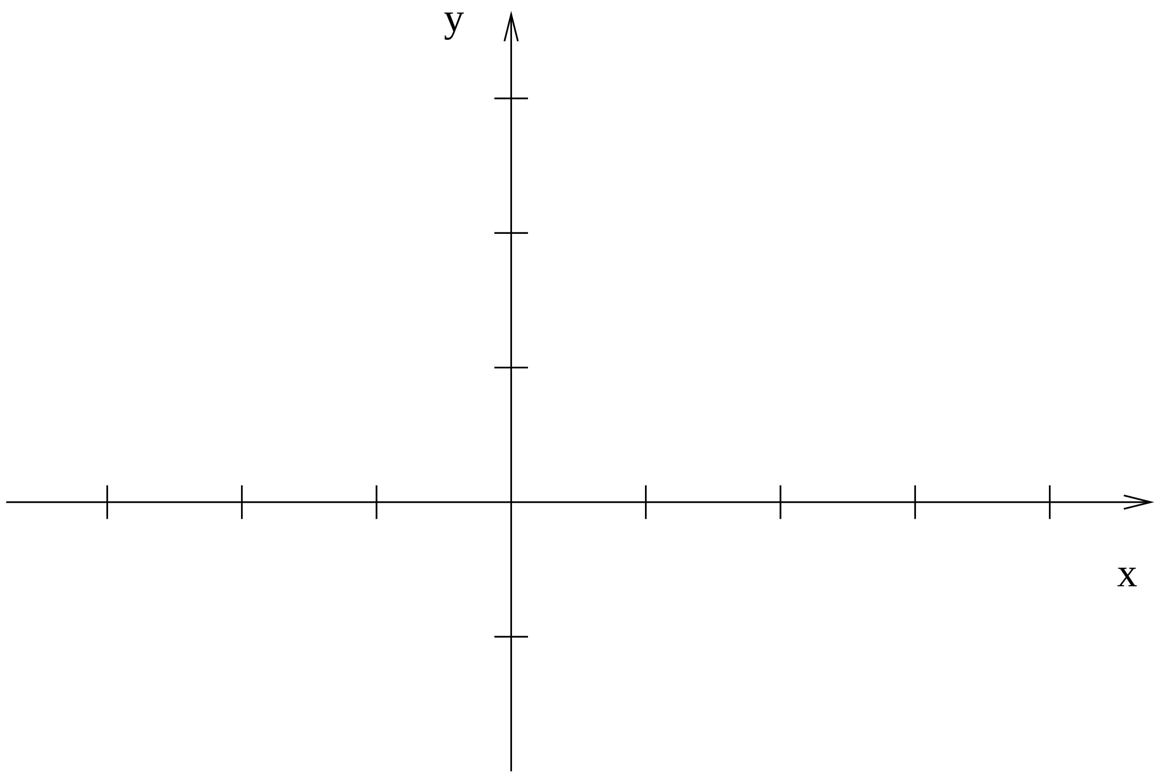
Problem 1. Consider the function f defined by

$$f(x) = \frac{1}{x}$$

Use the definition of the derivative function as a limit to find $f'(x)$.

Problem 2. Create a graph of a function f which satisfies the following 10 conditions:

1. $f(1) = f(-4) = 0$.
2. There is a local maximum at (0,3) (and no others).
3. There are local minima at (-2,1) and (1,0) (and no others).
4. $\lim_{x\to\infty} f(x) = 2$ and $\lim_{x\to-\infty} f(x) = -2$.
5. $f'(-4)$ and $f'(1)$ are undefined.
6. $f'(x) < 0$ on the intervals (-4,-2) and (0,1).
7. $f'(x) > 0$ on the intervals (-∞ , -4), (-2,0) and (1,∞).
8. There is an inflection point at $x = -1$.
9. $f''(x) < 0$ on the intervals (-1,1) and (1,∞).
10. $f''(x) > 0$ on the intervals (-∞, -4) and (-4,-1).



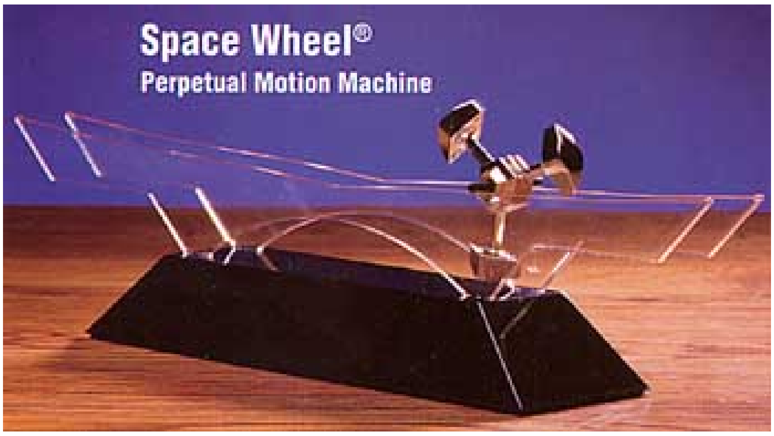
Problem 3. Consider the function h given by

$$h(x) = \frac{\cos(x) - 1}{\cos^2(x) + 1}$$

1. Write h as a composition of a rational function f and a trig function g.

2. Demonstrate the use of the chain rule by computing the derivative function $h'(x)$. The derivative from your calculator may be used as a "target", but will not gain you many points!

Problem 4. A "space wheel" rolls along a track on a cylindrical axle of radius r, rotating as it goes. It thus has two components of motion: its forward motion, and its rotation. The axle does not slip, but stays firmly in contact with the track.



1. Draw a picture that captures these two components of motion (using θ for the angle about the center of the axis, and s for the position of the axle along the track).

2. How is the rate of rotation related to the rate of forward motion along the track?

3. If a space wheel of radius 1/16 inches is rolling forward at a rate of 3 inches per second, how fast is it rotating (in radians per second)?

Problem 5. For the function

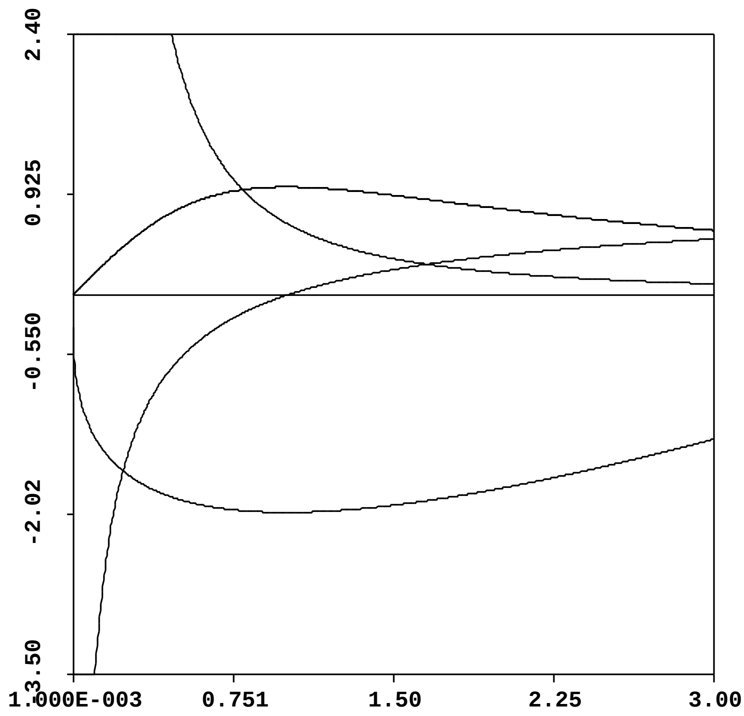
$$f(x) = \frac{x^2 - 1}{x + 2}$$

1. Use the first and second derivative test to determine where the function has local maximum and local minimum values.

2. Determine all intervals where the function is increasing.

3. Describe the concavity of the function. Are there any inflection points?

Problem 6. In the following plot we find the graphs of four functions. Identify the function f, and its first and second derivatives. The fourth function is unrelated to f. Explain your choices!



Problem 7. Verify that the function

$$f(x) = \frac{x}{x+2}$$

satisfies the hypotheses of the Mean Value Theorem (MVT) on the interval [1,4], then find all numbers c that satisfy the conclusion of the MVT.